Year 9 Term 3 Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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8 Year 9 Term 3 Week 8 Homework

8.1 The Linear Function

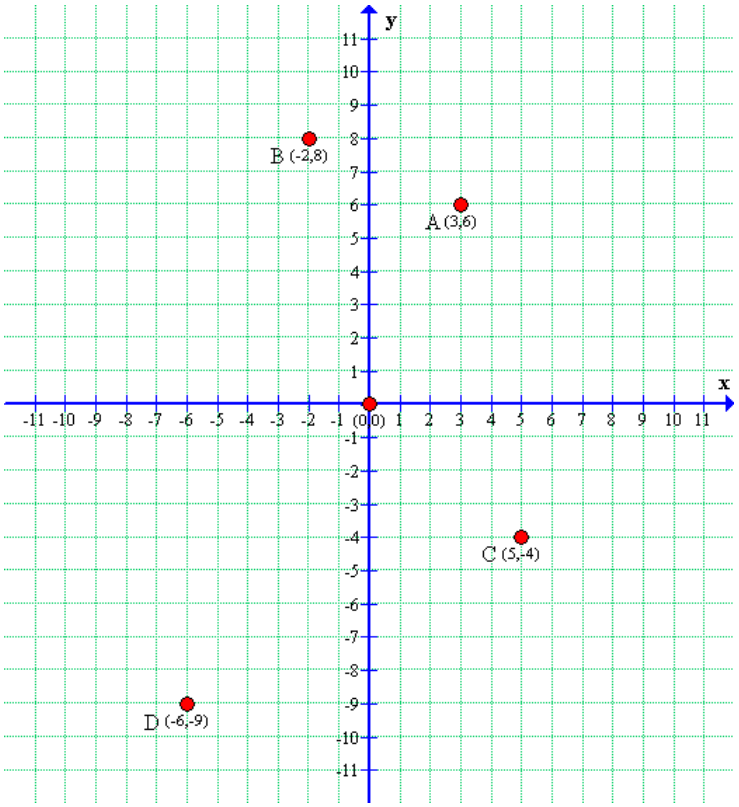
8.1.1 The number plane

We can plot a point on a number plane using a pair of numbers. This pair of numbers (written in brackets and separated by a comma) is called the **coordinates** of the point. Because the order in which these 2 numbers are written is important, it is also often called an **ordered pairs (x, y)**

- The first number always gives the number of units along the horizontal x-axis.
- The second number always gives the number of units up or down the vertical y-axis.

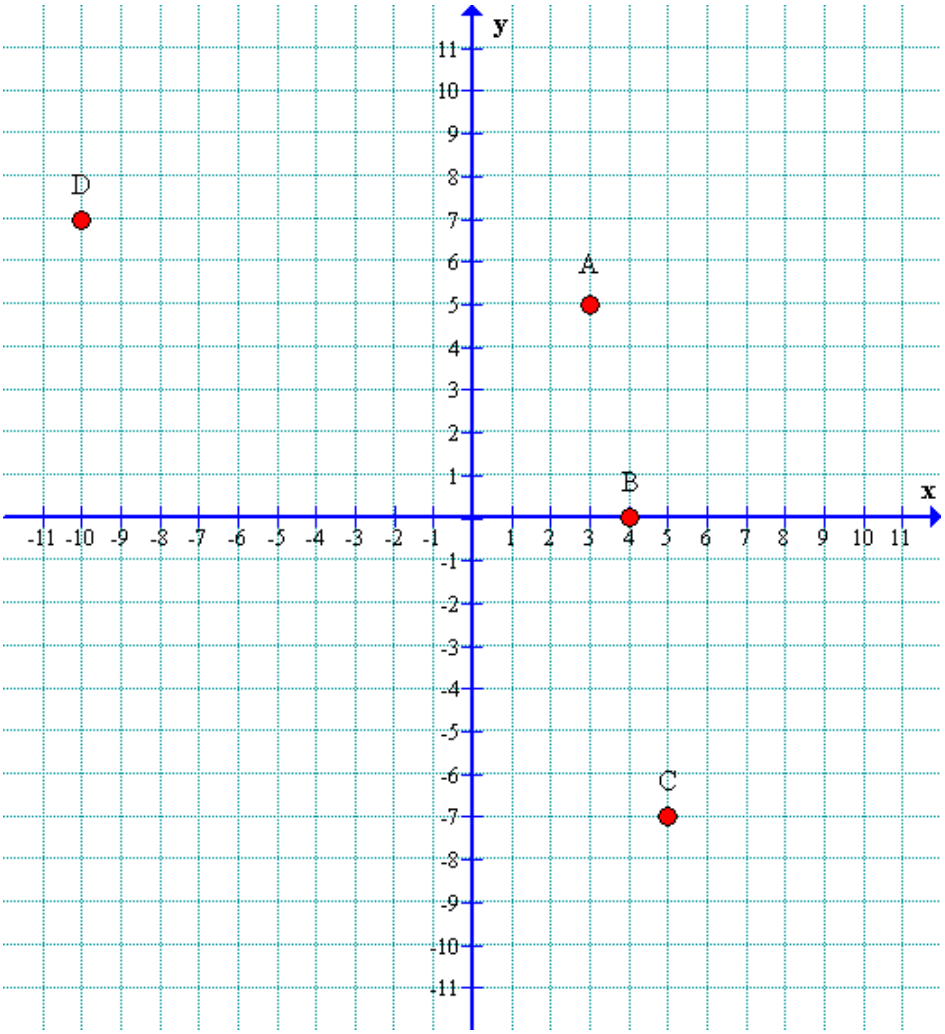
Example 8.1.1 Give the coordinates of the 5 points

Solution:



- *Coordinates of A are (3, 6).*
- *Coordinates of B are (-2, 8).*
- *Coordinates of C are (5, -4).*
- *Coordinates of D are (-6, -9).*
- *Coordinates of O are (0, 0) which is called the **Origin**.*

Exercise 8.1.1 Name the coordinates of the given points



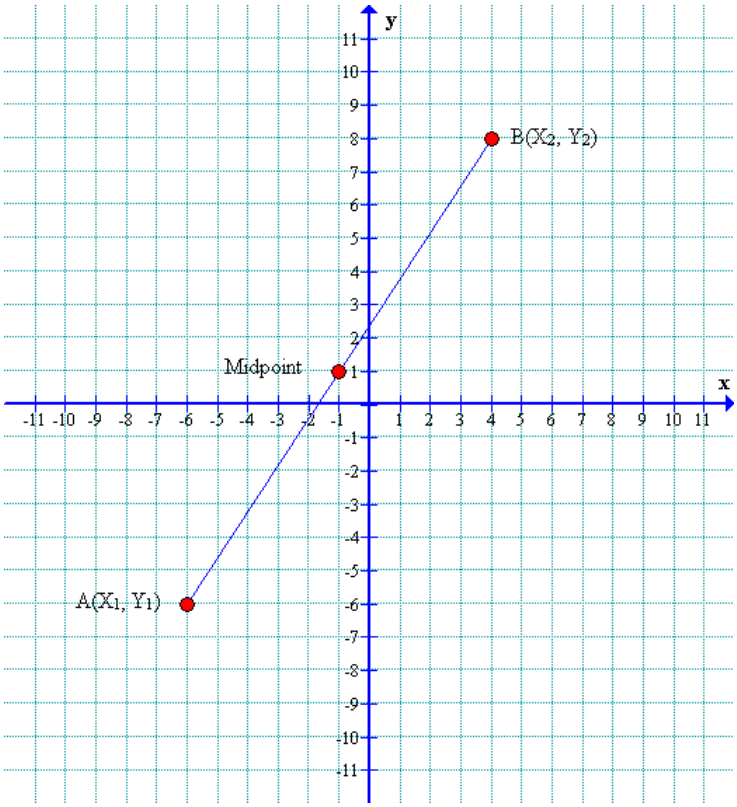
8.1.2 Midpoint formula

Consider a point A represented by coordinates (X_1, Y_1) and another point B represented by the coordinates (X_2, Y_2). Then the coordinates of the middle point between A and B can be calculated using the formula:

$$\left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2}\right)$$

It is really the average of the x coordinates, and the average of y coordinates.

Example 8.1.2 Calculate the midpoint between A and B.

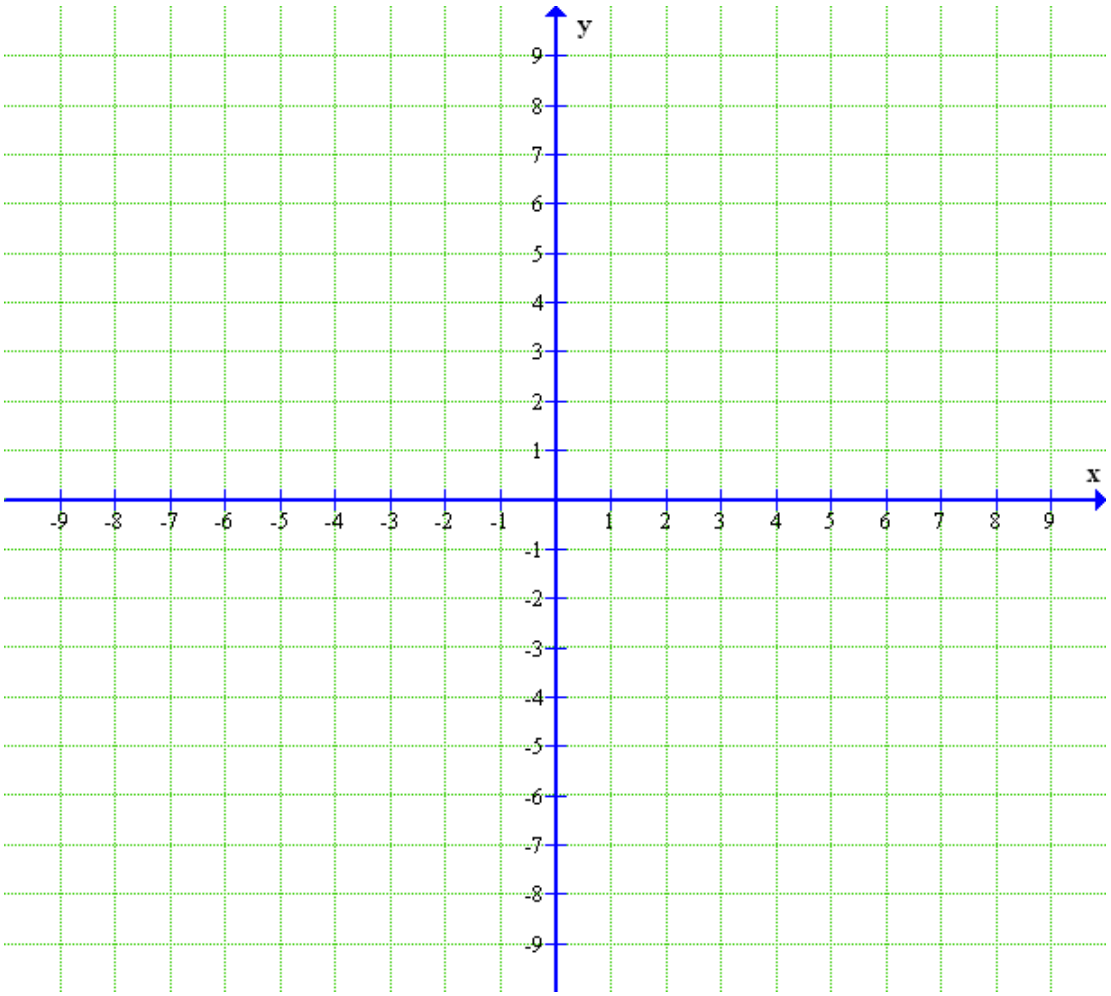


Solution:

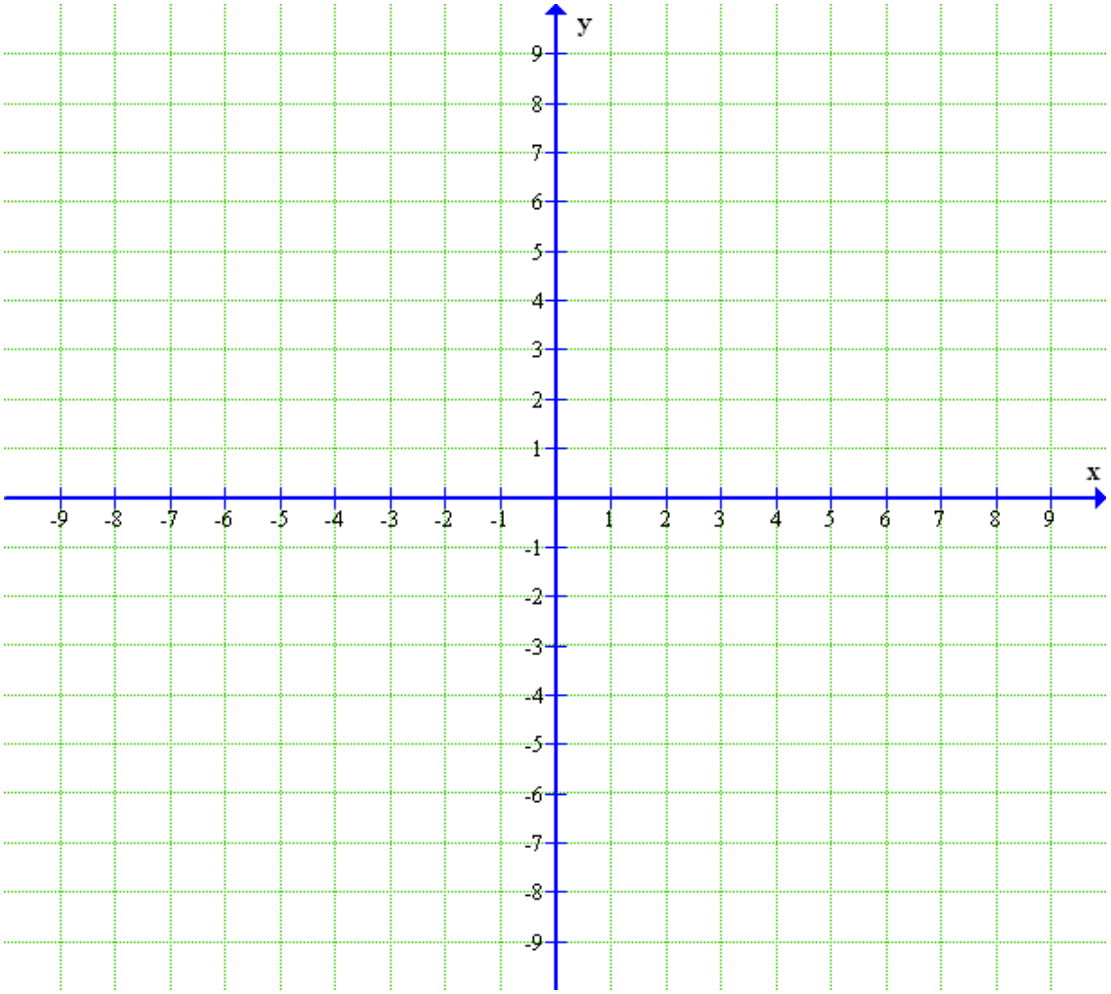
Let $(X_1,\ Y_1) = (-6, -6)$ and $(X_2,\ Y_2) = (4, 8)$

$$\begin{aligned} \therefore \text{Midpoint of AB} &= \left(\tfrac{X_1+X_2}{2}, \tfrac{Y_1+Y_2}{2}\right) \\ &= \left(\tfrac{-6+4}{2}, \tfrac{-6+8}{2}\right) \\ &= (-1, 1) \end{aligned}$$

Exercise 8.1.2 Joint the points $A(7,-3)$ and $B(0,7)$ and then find the midpoint of the interval joining:



Exercise 8.1.3 Joint the points $A(-5, 2)$ and $B(9, -4)$ and then find the midpoint of the interval joining:

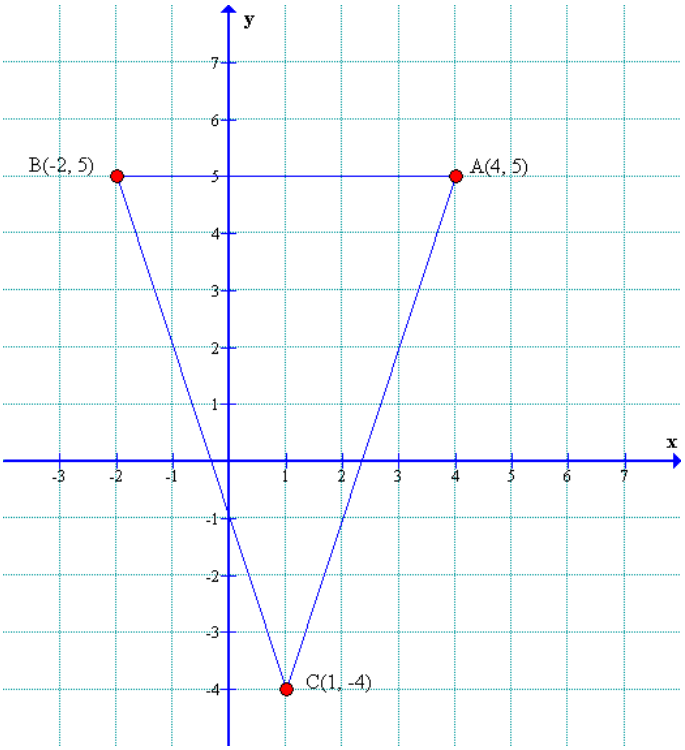


8.1.3 Distance between two points

If A has coordinates (x_1, y_1) and B has coordinates (x_2, y_2), then the straight line distance **d** between A and B is given by the formula shown below:

$$\mathbf{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 8.1.3 Show that the point $A(4, 5), B(-2, 5)$ and $C(1, -4)$ are the vertices of an isosceles triangle.



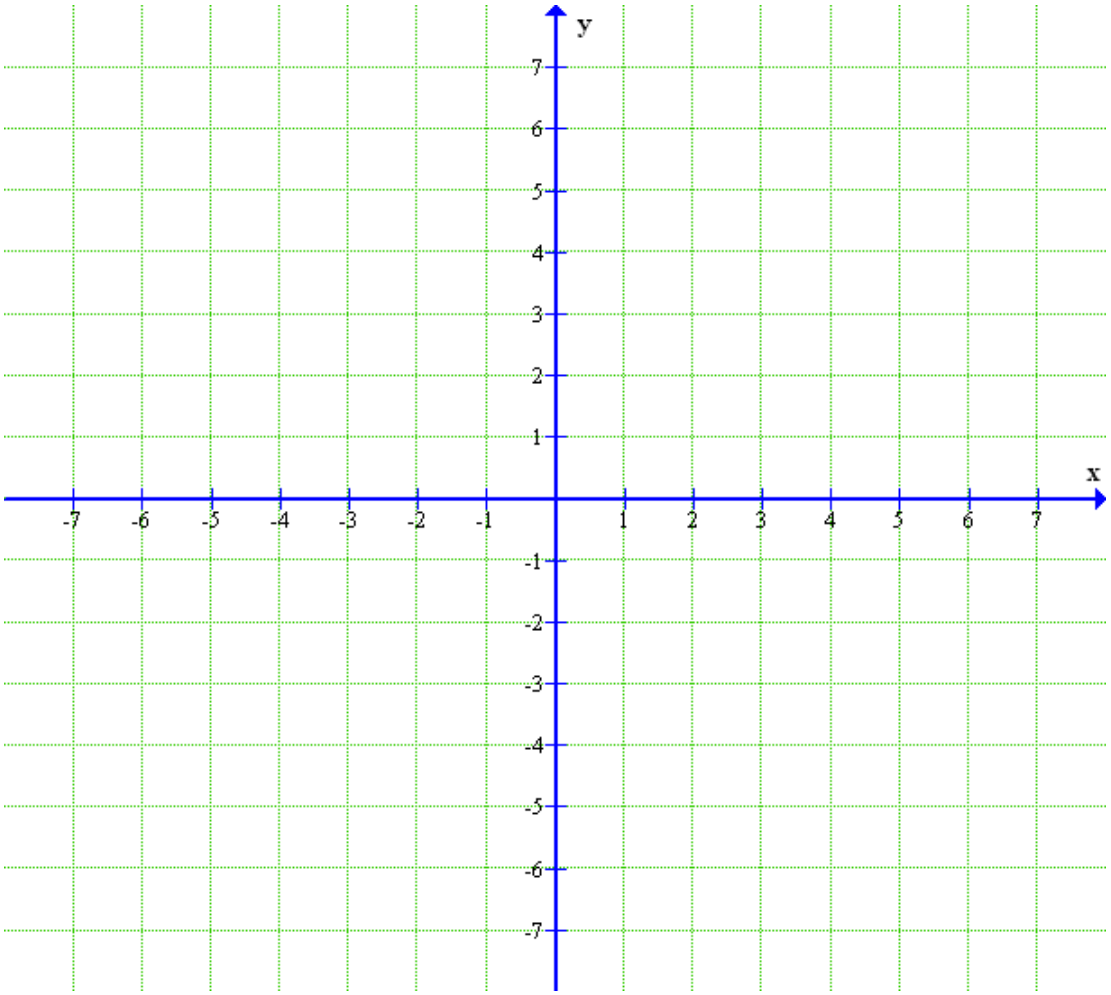
Solution:

In these type of questions, it is always best to start by drawing a quick sketch of the 3 points. From the diagram, it is now a simple matter to see that we have to prove that BC = AC by using the distance formula twice:

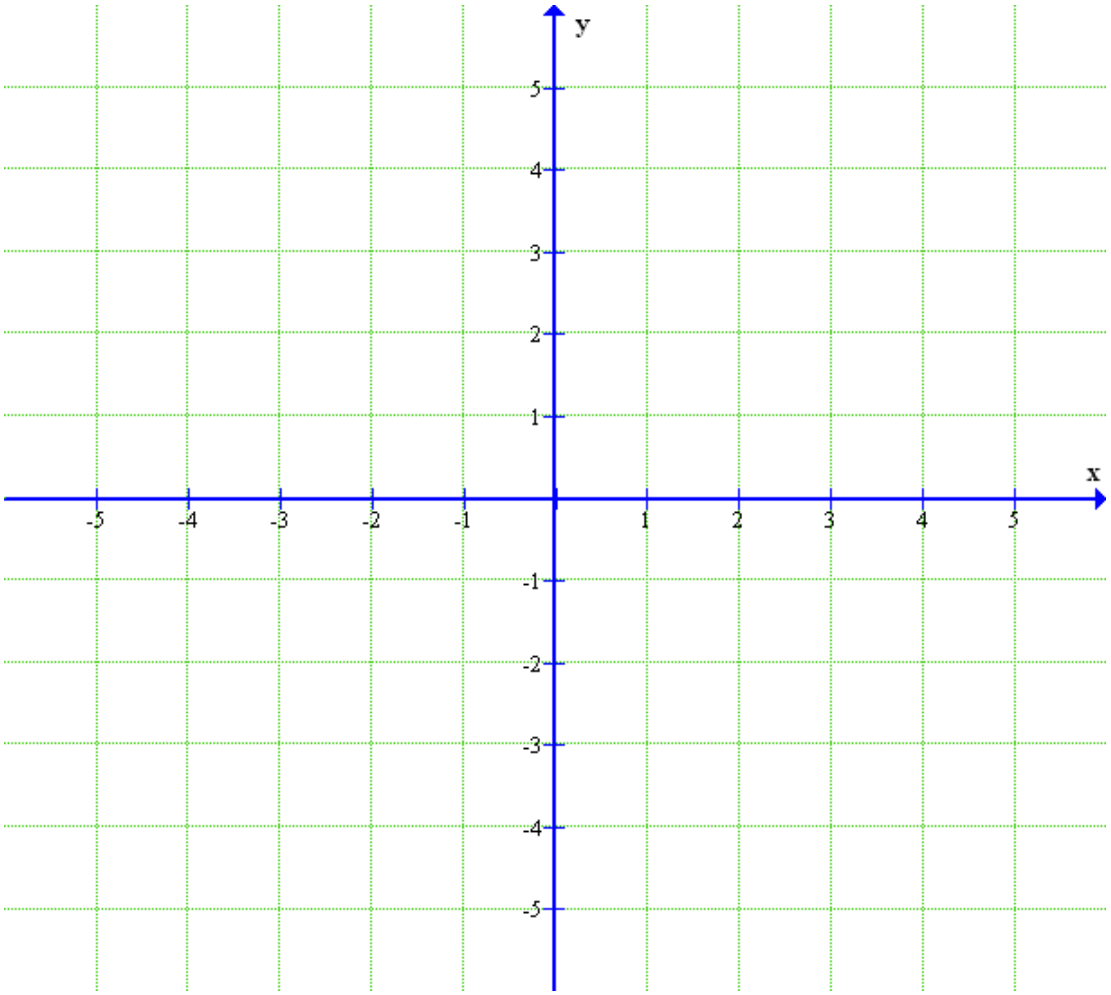
$$\begin{aligned} \text{BC} &= \sqrt{(1 - -2)^2 + (-4 - 5)^2} \\ &= \sqrt{3^2 + (-9)^2} \\ &= \sqrt{90} \end{aligned} \qquad \begin{aligned} \text{AC} &= \sqrt{(1 - 4)^2 + (-4 - 5)^2} \\ &= \sqrt{(-3)^2 + (-9)^2} \\ &= \sqrt{90} \end{aligned}$$

Since BC = AB = $\sqrt{90}$, therefore triangle ABC is found to be an isosceles.

Exercise 8.1.4 Show that the triangle whose vertices are $A(3,6), B(-2,-4)$ and $C(-5,2)$ is a right angled triangle and find the area of the triangle.



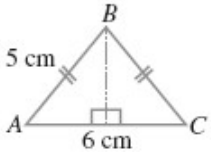
Exercise 8.1.5 Prove that a quadrilateral $ABCD$ with vertices $A(-1,4), B(-3,3), C(1,2)$ and $D(3,3)$ is a parallelogram.



8.2 Trigonometry

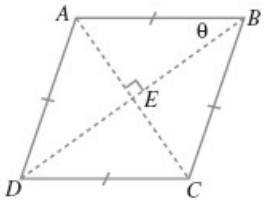
8.2.1 The trigonometry ratios

Exercise 8.2.1 In the isosceles $\triangle ABC$, $AB = BC = 5cm$ and $AC = 6$ cm. Find the values for:



1. *sin A =* _____
2. *cos A =* _____
3. *tan A =* _____

Exercise 8.2.2 A rhombus ABCD has diagonals AC and BD of length 16 cm and 30 cm respectively, intersecting at E.



1. *Find the side length of the rhombus.*

2. *Find the values for $sin\theta$, $cos\theta$ and $tan\theta$, where $\theta = \angle ABE$.*

$sin\theta$ = _____ , $cos\theta$ = _____ , $tan\theta$ = _____

8.2.2 Trigonometric ratios using a calculator

- To find the size of an angle given either a fraction or decimal:
  - press either the **INV**, **shift** or **2nd F** key followed by the appropriate trigonometric function key
  - enter the fraction or decimal into the calculator, then press = key
  - round off the angle as required.
- To find the length of the opposite or adjacent sides:
  - determine which ratio to be used
  - write down a trigonometric equation
  - multiply both sides by the denominator
  - then evaluate the value by using a calculator.

Exercise 8.2.3 Find the value of these trigonometric expressions, correct to 2 decimal places:

1. $sin\,35^\circ$ = _____
2. $tan\,48^\circ$ = _____
3. $cos\,12^\circ$ = _____
4. $\frac{25}{cos\,72^\circ}$ = _____
5. $\frac{1}{sin\,45^\circ}$ = _____
6. $\frac{1}{tan\,23^\circ}$ = _____
7. $\frac{52.2}{sin\,54^\circ}$ = _____
8. $\frac{1}{cos\,14.6^\circ}$ = _____

Exercise 8.2.4 Find the value of θ, correct to the nearest degree:

1. $sin\,\theta = 0.4384$ _____
2. $cos\,\theta = 0.9659$ _____
3. $tan\,\theta = 3.0624$ _____
4. $sin\,\theta = 0.9744$ _____
5. $cos\,\theta = 0.4225$ _____
6. $tan\,\theta = 5.7594$ _____

Exercise 8.2.5 Evaluate the following trigonometric expressions, correct to the 3 decimal places:

1. $\frac{10sin\,52^\circ}{5sin\,34^\circ}$ =

2. $\frac{sin\,45^\circ+cos\,18^\circ}{tan\,55^\circ}$ =

3. $\frac{tan\,69^\circ-sin\,24^\circ}{cos\,33^\circ}$ =

4. $\frac{sin\,45^\circ+cos\,72^\circ}{tan\,29^\circ\,tan\,50^\circ}$ =

5. $\frac{sin\,16^\circ\,cos\,40^\circ}{sin\,40^\circ\,cos\,16^\circ}$ =

Exercise 8.2.6 Further applications

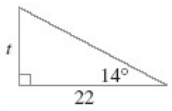
1. *If* $sin\,\theta = 0.7071$, *find* $cos\,\theta$ *and* $tan\,\theta$, *correct to 4 decimal places.*

2. *If* $cos\,\theta = 0.2588$, *find* $sin\,\theta$ *and* $tan\,\theta$, *correct to 4 decimal places.*

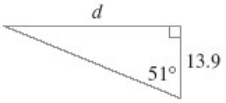
3. *If* $tan\,\theta = 0.3249$, *find* $sin\,\theta$ *and* $cos\,\theta$, *correct to 4 decimal places.*

Exercise 8.2.7 Use the trigonometric ratio to find the value of the pronumeral in each triangle, correct to 2 decimal places: (all measurements are in centimetres)

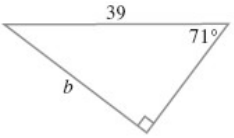
1. $t =$ _____



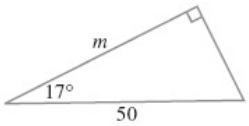
2. $d =$ _____



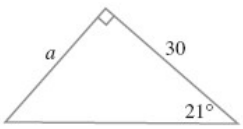
3. $b =$ _____



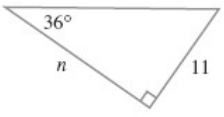
4. $m =$ _____



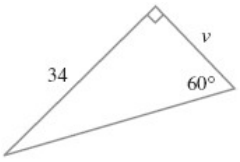
5. $a =$ _____



6. $n =$ _____



7. $v =$ _____

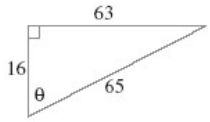


8.2.3 Problem solving

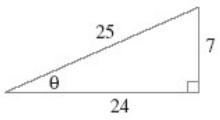
Exercise 8.2.8

1. *For each of the following right-angled triangles, state as a fraction the value of $sin\theta$, $cos\theta$ and $tan\theta$:*

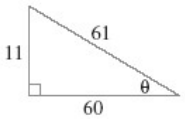
   (a) $sin\,\theta =$ _____ , $cos\,\theta =$ _____ , *and* $tan\,\theta =$ _____

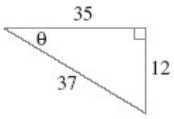
   (b) $sin\,\theta =$ _____ , $cos\,\theta =$ _____ , *and* $tan\,\theta =$ _____

   (c) $sin\,\theta =$ _____ , $cos\,\theta =$ _____ , *and* $tan\,\theta =$ _____

   (d) $sin\,\theta =$ _____ , $cos\,\theta =$ _____ , *and* $tan\,\theta =$ _____

2. *If $sin^2\,\theta$ means $(sin\,\theta)^2$, write down values for $sin^2\,\theta$ and $cos^2\,\theta$.*

   (a) $sin\,\theta$, = _____ , $cos\,\theta =$ _____ ,

   (b) Hence, find the value of $sin^2\,\theta + cos^2\,\theta$.

   _____

   _____

   _____

   (c) Repeat this question using any two triangles from the above question. What do you notice?

   _____

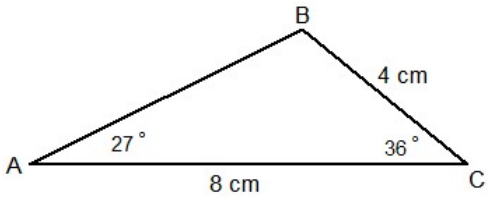
   _____

   _____

8.3 Maths Challenge

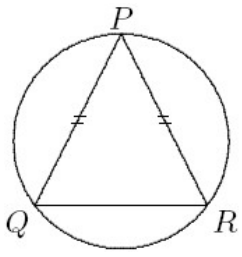
Exercise 8.3.1

1. *Find the area of the triangle shown below. correct to 1 decimal place.*



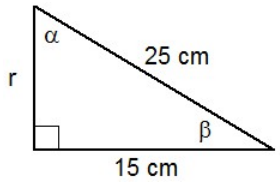
2. *Which value is bigger:* 2^{1000} *or* 3^{750}*? Explain.*

3. *PQR is an isosceles triangle with a base of 18 cm and sides of 15 cm and is inscribed in the circle shown. Find the radius of the circle, correct to 2 decimal places.*



8.4 Miscellaneous Exercise

Exercise 8.4.1 Find the value of:



1. r,

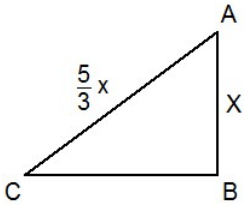
2. *the* $\angle\alpha$ *and* $\angle\beta$, *to the nearest degrees.*

3. $sin\,\alpha + cos\,\beta$,

4. $tan\,\alpha - tan\,\beta$,

5. $sin^2\,\alpha + cos^2\,\beta$

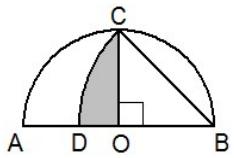
Exercise 8.4.2 For the diagram shown below, find:



1. *the length of BC in terms of x,*

2. *the size of $\angle ACB$, correct to the nearest minute.*

Exercise 8.4.3 The figure shown below is a semicircle of radius $r\ cm$ with centre 0. The radius OC is perpendicular to the diameter AB. An arc of a circle is drawn with centre B and radius BC, intersecting AB at D. Find:



1. *the length of OD in terms of r,*

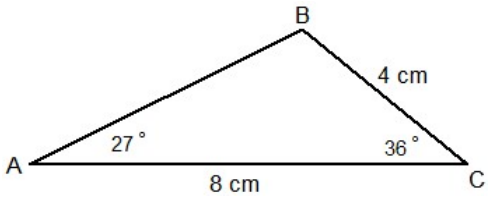
2. *the size of $\angle OBC$,*

3. *the area of the shaded region, leaving your answer in terms of r and π.*

8.5 Maths Challenge Solutions

Exercise 8.5.1

1. *Find the area of the triangle shown below. correct to 1 decimal place.*

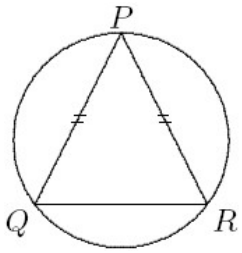


Solution: *Draw a line* $BD \perp AC$*; So* $BD = 4 \times sin36° = 2.35$ *cm;*
Area = $\frac{1}{2} \times b \times h$ = $\frac{1}{2} \times AC \times BD = 0.5 \times 8 \times 2.35 = 9.4\,cm^2$

2. *Which value is bigger:* 2^{1000} *or* 3^{750}*? Explain.*

Solution: $2^{1000} = 2^{4(250)} = 16^{(250)}$ *and* $3^{750} = 3^{3(250)} = 27^{250}$
Since $16 < 27$*, therefore* $16^{250} < 27^{250}$ *and* $2^{1000} < 3^{750}$

3. *PQR is an isosceles triangle with a base of 18 cm and sides of 15 cm and is inscribed in the circle shown. Find the radius of the circle, correct to 2 decimal places.*



Solution: *Draw a line* $PD \perp QR$*; So* $PD = \sqrt{15^2 - 9^2} = 12$
Let the radius of the circle be x.
$(12 - x)^2 + 9^2 = x^2$*;* $12^2 - 24x + x^2 + 91 = x^2$*;* $x = 9.375$ *cm = 9.38 cm.*