Year 9 Term 3 Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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This edition was printed on October 6, 2010.

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5 Year 9 Term 3 Week 5 Homework

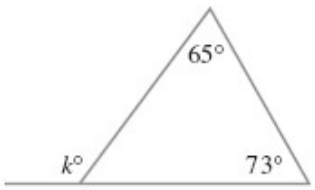
5.1 Geometry (Review)

5.1.1 Angle sum of a triangle

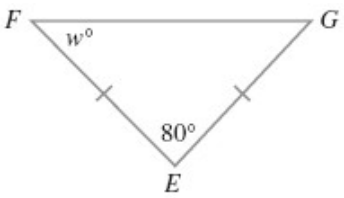
- The angle sum of a triangle is $180°$.
- The exterior angle of a triangle is equal to the sum of two interior opposite angles.
- In an equilateral triangle all angles are $60°$.
- In an isosceles triangle, the equal angles are opposite the equal sides.
- In any triangle, the longest side is opposite the largest angle, while the shortest side is opposite the smallest angle.

Exercise 5.1.1 Find the value of the pronumeral in each of the following figures:

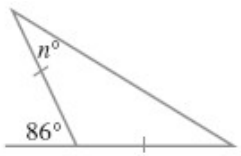
1. *Find the value of k.*



2. *If the $\triangle EFG$ is a isosceles triangle, find the value of w.*



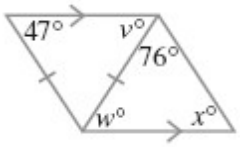
3. *Find the value of n.*



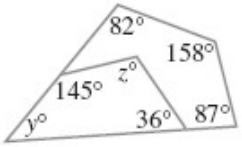
5.1.2 Angle sum of a quadrilateral

Exercise 5.1.2 Find the value of the pronumeral in each of the following figures, giving reasons.

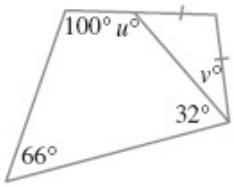
1. $x =$ _____, $v =$ _____ ,$w =$ _____



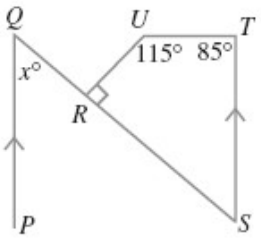
2. $y =$ _____, $z =$ _____



3. $u =$ _____ , $v =$ _____



4. $x =$ _____



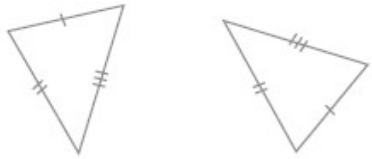
5.1.3 Tests for congruent triangles

Two triangles are said to be congruent if they have exactly the same size and shape. If two triangle are congruent, then:

1. the matching sides are equal in length
2. the matching angles are equal in size
3. the figures are equal in area.

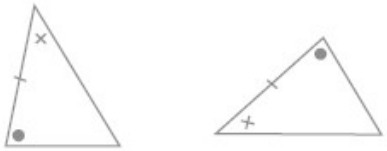
Tests:

- If the three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent (**SSS**).



- If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the two triangles are congruent (**SAS**).

- If two angles and one side of triangle are equal to two angles and the matching side of another triangle, then the two triangles are congruent (**ASA** or **AAS**).



- If the hypotenuse and a second side of one right-angled triangle are equal to the hypotenuse and a second side of another right-angled triangle, then the two triangles are congruent (**RHS**).

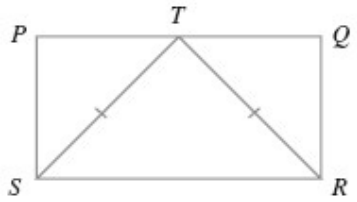
5.1.4 Congruence proofs

To prove that two triangles are congruent:

- identify the triangles that are being used in the proof
- name the three pairs of equal sides or angles
- name the congruent triangle given the vertices of the triangles in matching order and state the congruence test used.

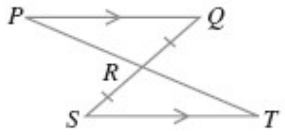
Example 5.1.1 PQRS is a rectangle and TS = TR, Prove that $\triangle TPS \equiv \triangle TQR$.

Solution: *in $\triangle TPS$ and $\triangle TQR$.*



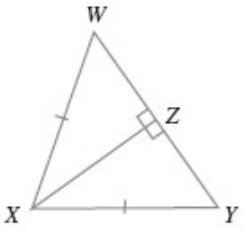
Solution: *TS = TR (given)*
$\angle TPS = \angle TQR = 90°$ *(All angles are right angles in a rectangle)*
PS = QR (opposite sides of a rectangle are equal)
$\therefore \triangle TPS \equiv \triangle TQR$ *(RHS)*

Exercise 5.1.3 Prove that $\triangle PQR \equiv \triangle TSR$.

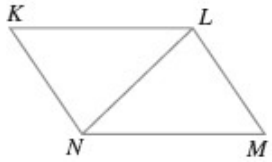


Exercise 5.1.4

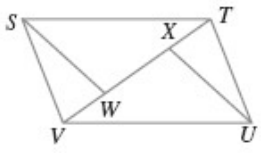
1. *In the $\triangle XYW$, $XW = XY$ and $XZ \perp WY$. Prove that $\triangle XWZ \equiv \triangle XYZ$.*



2. *KLMN is a parallelogram. Prove that $\triangle KLN \equiv \triangle MNL$.*



3. *STUV is a parallelogram. SW = XU and VW = XT. Prove that $\triangle SVW \equiv \triangle UTX$*

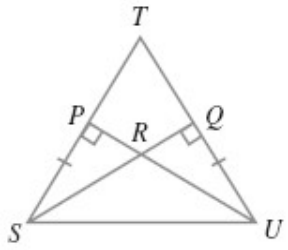


5.1.5 Deductive reasoning and congruent triangles

If one triangle is congruent to another, then the matching sides and angles in those triangle must be equal. The equivalence of these matching sides and angles can be used to prove that:

- two lines are parallel
- two lines are perpendicular
- a line bisects an interval
- a line bisects an angle
- a given triangle is isosceles
- a given quadrilateral is a parallelogram, a rectangle, a rhombus or a square.

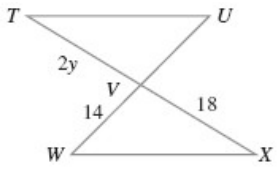
Example 5.1.2 If $\triangle TUS$ is an isosceles triangle, $UP \perp ST$ and $SQ \perp TU$, prove that the $\triangle PRS \equiv \triangle QRU$ and hence prove that $\triangle SRU$ is isosceles.



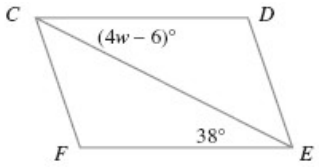
Solution: *In $\triangle PRS$ and $\triangle QRU$ PS = QU (given)*
$\angle RPS = \angle RQU = 90°$ *($UP \perp ST$ and $SQ \perp TU$)*
$\angle PRS = \angle QRU$ *(vertical opposite angles are equal)*
$\therefore \triangle PRS \equiv \triangle QRU$ *(AAS)*
$RS = RU$ *(matching side of congruent $\triangle s$)*
$\therefore \triangle SRU$ *is isosceles.*

Exercise 5.1.5 For each of the following prove that the two triangles are congruent, hence find the value of the pronumeral.

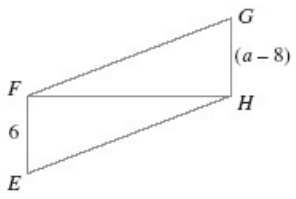
1. *$TU||WX$ and $TU = WX$. Prove $\triangle TUV \equiv \triangle VWX$*



2. *$CF||DE$ and $CF = DE$. Prove that $\triangle CEF \equiv \triangle CDE$*



3. *$EF \perp FH$, $GH \perp FH$ and $EF = GH$. Prove that $\triangle GFH \equiv \triangle EFH$*



5.1.6 Problem solving

Exercise 5.1.6

1. *Is a rhombus a regular polygon? Why or why not?*

2. *Calculate the interior angle sum of each of these regular polygons, then find the size of their interior angles:*

   (a) *decagon*

   (b) *dodecagon*

3. *Find the size of the exterior angles in a regular:*

   (a) *pentagon*

   (b) *nonagon*

4. *How many sides has a regular polygon with:*

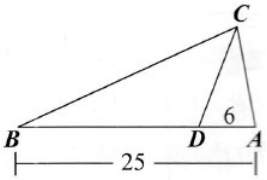
   (a) *exterior angles measuring* 15°

   (b) *interior angles measuring* 175°

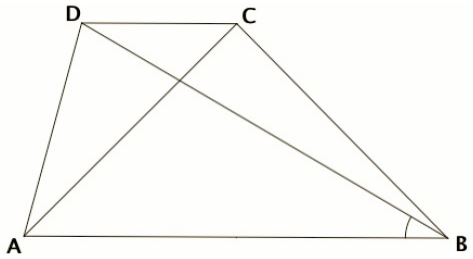
5.2 Math challenge

Exercise 5.2.1

1. *The area of $\triangle ABC$ is 50 cm^2. Point D lies on side BA, $DA = 6$ cm and $BA = 25cm$. Find the area of $\triangle CBD$.*



2. *In the following trapezium, $DC//AB$, $AC \perp CB$, $AC = CB$ and $BA = BD$. Find the $\angle ABD$.*



3. *In a magic square the rows, columns and diagonals each add up to the same number. In this magic square the numbers 39, 47, 55, 63 67 and 71 are missing. What number does $\boxed{?}$ stand for?*

43		59
		?
51		

5.3 Miscellaneous exercise

Exercise 5.3.1

1. *The exterior angles of a pentagon are* $2x°$, $(2x + 3)°$, $(3x + 12)°$, $(3x - 20)°$ *and* $(x + 35)°$. *Calculate the following:*

   (a) the value of x,

   (b) the smallest interior angle,

   (c) the smallest exterior angle.

2. *Evaluate the following:*

   (a) $6 \times [108 \div (-17 + 8)] - 36 + 12$

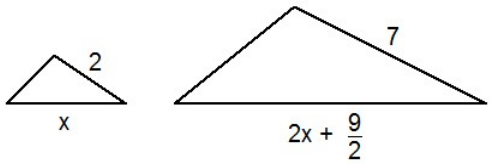
   (b) $(2\frac{1}{3} + 1\frac{2}{9}) \times 2\frac{2}{5}$

   (c) $12^2 - 2^4 + 7^2$

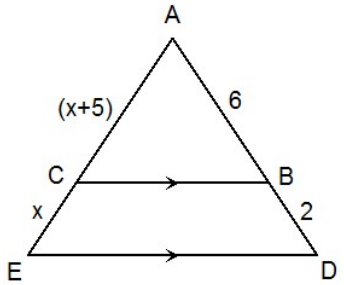
3. *A clock gains 12 minutes 15 seconds in one week. How many days will it take to gain 1 hour and 45 minutes?*

Exercise 5.3.2

1. *Given that the two triangles are similar, form an equation in terms of x and solve it.*



2. *In the diagram shown below, BC is parallel to ED. $AB = 6$ cm, $BD = 2$ cm, $AC = (x + 5)$ cm and $CE = x$ cm. Find the value of x.*



3. *Given that $-3 \leq x \leq 4$ and $2 \leq y \leq 6$, find the following:*

   (a) *the least possible value of $x - y$.*

   (b) *the least possible value of xy.*

Exercise 5.3.3

1. *If the solution of* $x^2 + bx + 36 = 0$ *are integers, find the number of integer values b can have.*

2. *There are exactly 3 integers x satisfying the inequality* $x^2 + bx + 2 \leq 0$*. How may integer value of b are possible?*

3. *If* $\sqrt{2} = 1 + \frac{1}{2+\frac{1}{2+x}}$*, find the value of x.*

4. *How many real solution of the equation* $\sqrt{x^2 + \sqrt{x^3 + 1}} + x = 1$ *can have?*

5. *A number is added to one third of itself. The result is 108. What is the number?*

Exercise 5.3.4 The rhombus $ABCD$ has a perimeter of 68 cm. The diagonals AC and BD intersect at P and $\angle ABC = 68°$.

1. *Find the size of $\angle ABP$.*

2. *Find the length of the diagonal AC, correct to one decimal place.*

Exercise 5.3.5

1. *A chord AB of length $16\,cm$ is drawn in a circle with centre O and area $100\pi\,cm^2$. Find the size of the angle that the chord AB subtends at the centre of the circle (correct to nearest minutes).*

2. *$\triangle ABC$ is an equilateral triangle with sides $12\,cm$ long. D is a point on BC, $4\,cm$ from B. Find the size of $\angle ADC$ (correct to nearest minutes).*

3. *Find the value of x correct to one decimal place.*