Year 9 Term 2 Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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10 Year 9 Term 2 Week 10 Homework

10.1 Surds

10.1.1 Adding and subtracting surds

Exercise 10.1.1

1. $8\sqrt{2} + \sqrt{72}$ = _____
2. $2\sqrt{63} + 5\sqrt{7}$ = _____
3. $2\sqrt{96} + 6\sqrt{6}$ = _____
4. $6\sqrt{13} + 2\sqrt{52}$ = _____
5. $13\sqrt{6} - 2\sqrt{150}$ = _____
6. $13\sqrt{11} - 6\sqrt{44}$ = _____
7. $3\sqrt{98} - 2\sqrt{32}$ = _____
8. $4\sqrt{45} - 5\sqrt{20}$ = _____
9. $\sqrt{4ab^2} \times \sqrt{ab^2}$ = _____
10. $2\sqrt{12m} \times 3\sqrt{18m}$ = _____

10.1.2 Multiplication and division of surds

Exercise 10.1.2

1. $\sqrt{10} \times \sqrt{2}$ = _____
2. $2\sqrt{3} \times 3\sqrt{6}$ = _____
3. $\sqrt{5} \times \sqrt{7} \times 2\sqrt{3}$ = _____
4. $3\sqrt{5} \times 2\sqrt{7} \times 8\sqrt{2}$ = _____
5. $15\sqrt{21} \div 3\sqrt{3}$ = _____
6. $4\sqrt{28} \div 2\sqrt{2}$ = _____
7. $32\sqrt{65} \div 8\sqrt{5}$ = _____
8. $21\sqrt{abc} \div 3\sqrt{ac}$ = _____
9. $4\sqrt{p^3q^2} \div 2\sqrt{pq}$ = _____
10. $18\sqrt{75a^5} \div 3\sqrt{3a}$ = _____

10.1.3 Binomial products with surds

The rules for expansion of binomial products can be used to expressions that involve surds:

- $a(b+c) = ab + ac$
- $(a+b)(c+d) = ac + ad + bc + bd$
- $(a+b)(a-b) = a^2 - b^2$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$

Example 10.1.1 Expand and simplify each of the following:

1. $\sqrt{3}(\sqrt{3}+5)$ = $3 + 5\sqrt{3}$
2. $2\sqrt{3}(3\sqrt{2}+\sqrt{5})$ = $6\sqrt{6} + 2\sqrt{15}$
3. $\sqrt{3}(\sqrt{12}-\sqrt{6})$ = $6 - 3\sqrt{2}$
4. $(2\sqrt{5}+3\sqrt{10})(2\sqrt{5}-3\sqrt{10})$ = $(2\sqrt{5})^2 - (3\sqrt{10})^2$ = *20 - 90 = -70*

Exercise 10.1.3 Expand and simplify each of the following:

1. $(5+\sqrt{7})(5-\sqrt{7})$

2. $(\sqrt{7}-3)(\sqrt{7}+3)$

3. $(2\sqrt{5}+\sqrt{2})(2\sqrt{5}-\sqrt{2})$

4. $(\sqrt{2}-3)(\sqrt{10}+4)$

5. $(2\sqrt{3}-5)(2\sqrt{3}+5)$

Exercise 10.1.4 Consolidation

1. $(3\sqrt{2}-2\sqrt{3})(5\sqrt{6}-3\sqrt{8})$

2. $(2\sqrt{5}-4\sqrt{2})(3\sqrt{8}-5\sqrt{10})$

3. $(\sqrt{12}+\sqrt{10})(\sqrt{3}-\sqrt{2})$

4. $(5\sqrt{3}-\sqrt{2})(2\sqrt{7}+\sqrt{3})$

5. $(2\sqrt{5}-\sqrt{11})(2\sqrt{5}+\sqrt{3})$

6. $(2\sqrt{3}-\sqrt{7})^2$

Exercise 10.1.5 Further applications

1. $(2\sqrt{5}+\sqrt{3})(2\sqrt{5}-\sqrt{3})$

2. $(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})$

3. $(3\sqrt{6}+5\sqrt{2})(3\sqrt{6}-5\sqrt{2})$

4. *Find the value of m and n in each of these, where m>0.*

   (a) $(m+\sqrt{n})^2 = 14+6\sqrt{5}$

   (b) $(\sqrt{m}+n)^2 = 43-12\sqrt{7}$

10.1.4 Rationalising the denominator

Fractions that have a surd in the denominator need to be rationalised:

- To rationalise a monomial denominator, multiply the numerator and denominator by the surd in the denominator (as $\sqrt{a} \times \sqrt{a} = a$).
- Simplify the surd in the numerator if possible.
- To rationalise a binomial denominator, multiply the numerator and denominator by the conjugate of the denominator. (The conjugate of the binomial (a+b) is (a-b)).
- Simplify the surd in the numerator if possible.

Example 10.1.2

1. $\frac{6}{3\sqrt{5}} = \frac{6}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
   $= \frac{6\sqrt{5}}{15}$
   $= \frac{2\sqrt{5}}{5}$
2. $\frac{\sqrt{3}+2}{\sqrt{3}-2} = \frac{\sqrt{3}+2}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$
   $= \frac{(\sqrt{3}+2)^2}{3-4}$
   $= \frac{3+4\sqrt{3}+4}{-1}$
   $= -7 - 4\sqrt{3}$

Exercise 10.1.6 Rationalise the denominator in each of these:

1. $\frac{\sqrt{3}}{\sqrt{6}}$

2. $\frac{\sqrt{7}+3}{\sqrt{2}}$

3. $\frac{1}{\sqrt{5}-2}$

Exercise 10.1.7 Express each fraction in simplest form with a rational denominator.

1. $\frac{2\sqrt{3}}{3\sqrt{2}}$

2. $\frac{5\sqrt{3}}{2\sqrt{5}}$

3. $\frac{3\sqrt{6}}{\sqrt{2}}$

4. $\frac{1+\sqrt{3}}{\sqrt{2}}$

5. $\frac{\sqrt{3}+1}{2\sqrt{3}}$

6. $\frac{3+\sqrt{3}}{\sqrt{2}-1}$

10.2 Miscellaneous exercises

Exercise 10.2.1

1. *Solve the equation* $x + \frac{1}{x} = 3$. *Leave answers in surd form.*

2. *Express* $\frac{4+\sqrt{3}}{2+\sqrt{3}}$ *as a fraction with a rational denominator.*

3. *Express* $\sqrt{45} + \sqrt{80}$ *in the form* $a\sqrt{5}$ *and hence find the* a.

4. *Express* $\frac{4+\sqrt{3}}{2+\sqrt{3}}$ *in the form* $b - 2\sqrt{3}$ *and find* b.

5. *Find the discriminant of the following equation and state whether the roots are real.*

$$2x^2 + 3x + 5 = 0$$

10.3 Maths challenge

Exercise 10.3.1

1. *Given that k is a positive number specify, find the smallest and the largest of the following numbers:* $2^{-\frac{1}{2}k},\ 2^{\frac{1}{2}k},\ 2^{k},\ 2^{-k}$.

2. *By expressing* $\frac{4}{2+\sqrt{5}} - \frac{1}{9-4\sqrt{5}}$ *in its simplest form show that it is a rational number.*

3. *Express* $\frac{x+1}{x^2-x} - \frac{x-1}{x^2+x}$ *as a fraction in its lower terms.*

4. *Given that x = 3 is one of the root of the quadratic equation* $mx^2 - 20x + m = 0$*, find the exact value of the other root.*

5. *For all values of* $\boldsymbol{k}$ *for which the quadratic equation* $kx^2 - 8x + k = 0$ *has real roots.*