Year 7 Term 1 Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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4 Year 7 Term 1 Week 4 Homework

4.1 Number theory

4.1.1 Odd and Even Numbers

Exercise 4.1.1 Write down the following with no numerals repeated:

1. *smallest 3-digit odd number with 7 in the tens place* _____
2. *largest 3-digit even number with 5 in the tens place* _____
3. *smallest 3-digit odd number with a 4 in the hundreds place and 5 in the tens place* _____
4. *largest 4-digit even number with a 6 in the thousands place and a 3 in the tens place* _____
5. *smallest 4 digit odd number with a 2 in the tens place* _____

4.1.2 Special Numbers

- Palindromic numbers: a palindrome is a number that reads the same from left to right or from right to left.
  For example, 171, 34543 are palindromic numbers.
- Square numbers: 1, 4, 9, 16, 25, . . . are square numbers.
- Triangular numbers: The pattern of triangular number is derived from the following diagrams.

Note that each triangle has one more row than the previous triangle and each row has one more than the row above. The number of stars in each triangle can be found by adding consecutive counting number. That is $1+2=3, 1+2+3=6, 1+2+3+4=10$

Exercise 4.1.2

1. *Find the 8th square number* _____
2. *Find the 12th square number* _____
3. *Find the 9th triangular number* _____

Exercise 4.1.3

1. *What is the smallest 3-digit palindromic number?* _____
2. *What is the largest 3-digit palindromic number?* _____
3. *What is the 13th square number?* _____

Exercise 4.1.4 Further applications

1. *How many consecutive odd numbers would have to be added to find the value of* 14×14 *?*

2. *Find the first two square numbers that are both the sum of two other square numbers.*

3. *Find the first four square numbers that are each the product of two different square numbers other than 1.*

4. *Find the first triangular number that is equal to the product of two other triangular numbers.*

5. *Find the first four triangular numbers that are equal to the sum of two other triangular numbers.*

6. *Find the first square number that is divisible by 12.*

7. *Find the first odd 2-digit square number.*

4.1.3 Fibonacci Numbers

The sequence of Fibonacci numbers begins with two consecutive 1s and each term thereafter is found by adding the two previous terms.

For example: 1, 1, $1+1=2$, $1+2=3$, $2+3=5$, $3+5=8$, $5+8=13$, $8+13=21$, . . .
$[1, 1, 2, 3, 5, 8, 13, 21, ...]$.

Exercise 4.1.5

1. *Find the first 10 Fibonacci numbers.*

2. *Find the first Fibonacci number, other than 1, that is are also a square number.*

3. *Which of the first 3 Fibonacci numbers, other than 1 are also triangular numbers?*

4. *Will the 25th Fibonacci number be odd or even?*

5. *Will the 30th fibonacci number be odd or even?*

6. *If the 13th fibonacci number is 233, what is the sum of the first 11 Fibonacci numbers?*

7. *Find the sum of the first 10 Fibonacci numbers. Is it divisible by 11?*

4.1.4 Divisibility Tests

The following test can be used to determine whether one number is divisible by another without performing the division:

- A number is divisible by 2 if it an even number.
- A number is divisible by 3 if the sum of the digits is divisible by 3.
- A number is divisible by 4 if the number formed by the last two digits is divisible by 4.
- A number is divisible by 5 if the last digit is either 5 or 0.
- A number is divisible by 6 if the number is divisible by both 2 and 3.
- A number is divisible by 8 if the number formed by the last three digits is divisible by 8.
- A number is divisible by 9 if the sum of the digits is divisible by 9.
- A number is divisible by 11 if the sum of the digits in the odd position is equal to the sum of digits in the even positions, or they differ by a number that is divisible by 11.

Exercise 4.1.6 Use the divisibility tests to show that:

1. *978 is divisible by 6* _____
2. *5848 is divisible by 8* _____
3. *1524 is divisible by 4* _____
4. *289564 is divisible by 11* _____
5. *786 is divisible by 3* _____
6. *9567 is divisible by 9* _____

Exercise 4.1.7 Find the sum of the digits and hence determine which numbers are divisible by 3.

1. *252* _____
2. *445* _____
3. *468* _____
4. *2989* _____
5. *20593* _____

Exercise 4.1.8 Find the sum of the digits and hence determine which numbers are divisible by 9.

1. *516*
2. *648*
3. *5832*
4. *9972*
5. *14258*

Exercise 4.1.9 Look at the sum of the digits in the odd positions and the sum of the digits in even positions. Hence determine which numbers are divisible by 11.

1. *275*
2. *2166*
3. *17358*
4. *71258*
5. *93523*

Exercise 4.1.10 To test a number for divisibility by 6, we test for divisibility by both 2 and 3. How could we test a number for divisibility by:

1. *12 ?*
2. *14 ?*
3. *45?*

Exercise 4.1.11

1. *Is 795 divisible by 15?*
2. *Is 518 divisible by 12?*
3. *Is 126 divisible by 14?*

4.1.5 Problem solving

Exercise 4.1.12 The pages of a certain book are numbered consecutively from 1 to 500. How many page numbers meet each of the following sets of conditions?

1. *The page numbers contain the digit 5 and are also divisible by 5.*

2. *The page numbers contain the digit 5 and are not divisible by 5.*

3. *The page numbers do not contain the digit 5 but are divisible by 5.*

Exercise 4.1.13 Suppose that each if the following numbers are added. Is the sum even or odd?

1. *four even numbers*
2. *four odd numbers*
3. *one even number and one odd number*
4. *two even numbers and two odd numbers*
5. *three even numbers*
6. *three odd numbers*
7. *five odd numbers and one even number*
8. *9 even numbers and 2 odd numbers*
9. *ninety-nine odd numbers and one even number*
10. *thirty-nine even numbers and twenty odd numbers*

Exercise 4.1.14 Find the missing digit so that the resulting number is divisible by 3.

1. *1234?95*
2. *238?4*
3. *6?54321*
4. *?649*

Exercise 4.1.15 Find the missing digit so that the resulting number is divisible by 9.

1. *1324?57*
2. *318?4*
3. *6?5321*
4. *?6249*

Exercise 4.1.16 Find all possible values for the missing digits in ?5? so the the resulting number is divisible by 9.

Exercise 4.1.17 Consider the following statement.'If two numbers are each divisible by another number then their sum is also divisible by that number.'

1. *Is the statement true or false?*
2. *Give two examples:*

4.2 Diagnostic Test

1. Find the value of $\frac{7!}{9!}$. [5]

2. Find the value of $\frac{5!+6!}{7!}$. [5]

3. I am thinking of a number. When I multiply it by 5 and subtract the answer from its square, I get 14. What is the number? [5]

4. Find the two square numbers whose difference is 17. [5]

5. Find the first square number whose digits differ by 2. [5]

6. Find the first 2-digit square number whose digits are even. [5]

7. Two triangular numbers differ by 23. If the smallest of the numbers is 253, what is the other number? [5]

8. Find the first 2-digit triangular number whose digits are also triangular numbers. [5]

9. If the sum of the first 12 Fibonacci number is 376, what is the 14th Fibonacci number? [10]

10. Find the missing digit so that the resulting number is divisible by 3. 124?46 [10]

11. Find the missing digit so that the resulting number is divisible by 9. 3475?28 [10]

12. Find the missing digit so that the resulting number is divisible by 11. 317?82 [10]

13. A four-digit number contains the digit 1, 4, 7, 9, but not in that order. arrange these digits so that the resulting number is divisible by 11. In how many ways can this be done? [10]

14. I am thinking of a 5 digit number which is divisible by both 4 and 6. If I divided this number by 12, what would be the remainder be? [10]