

## 2/3 Unit Math Homework for Year 12

<b>Student Name:</b> _____	<b>Grade:</b> _____
<b>Date:</b> _____	<b>Score:</b> _____

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## 2 Integration Part 2 Definite Integral

### 2.1 Rules of Differentiation and Integration

$f(x)$	$f'(x)$	$f(x)$	$\int f(x).dx$
$kx$	$k$	$k$	$kx + c$
$x^n$	$nx^{n-1}$	$x^n$	$\frac{x^{n+1}}{n+1} + c$
$ax^n + bx + c$	$anx^{n-1} + b$	$ax^n$	$\frac{ax^{n+1}}{n+1} + c$
$f(h(x))$	$f'(h(x)).h'(x)$	$(ax + b)^n$	$\frac{(ax+b)^{n+1}}{a(n+1)} + c$
$g(x).h(x)$	$g'(x).h(x) + g(x).h'(x)$	$f'(x).(f(x))^n$	$\frac{1}{n+1}(f(x))^{n+1} + c$
$y = uv$	$y' = uv' + u'v$		
$\frac{g(x)}{h(x)}$	$\frac{g'(x).h(x) - g(x).h'(x)}{[h(x)]^2}$		
$y = \frac{u}{v}$	$y' = \frac{u'v - uv'}{v^2}$		
$e^x$	$e^x$	$e^x$	$= e^x + c$
$e^{kx}$	$ke^{kx}$	$e^{kx}$	$\frac{1}{k}e^{kx} + c$
$\ln x$	$\frac{1}{x}$	$\frac{1}{x}$	$\ln x + c$
$\ln kx$	$\frac{k}{x}$	$\frac{k}{x}$	$\frac{1}{k} \ln x + c$
$\ln f(x)$	$\frac{f'(x)}{f(x)}$	$\frac{f'(x)}{f(x)}$	$\ln f(x) + c$
$\sin x$	$\cos x$	$\sin x$	$-\cos x + c$
$\sin kx$	$k \cos kx$	$\sin kx$	$-\frac{1}{k} \cos kx + c$
$\cos x$	$-\sin x$	$\cos x$	$\sin x + c$
$\cos kx$	$-k \sin kx$	$\cos kx$	$\frac{1}{k} \sin kx + c$
$\tan x$	$\sec^2 x$	$\sec^2 x$	$\tan x + c$
$\tan kx$	$k \sec^2 kx$	$\sec^2 kx$	$\frac{1}{k} \tan kx + c$
$\sin^{-1} \frac{x}{a}$	$\frac{1}{\sqrt{a^2 - x^2}}$	$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1} \frac{x}{a} + c$
$\cos^{-1} \frac{x}{a}$	$-\frac{1}{\sqrt{a^2 - x^2}}$		
$\tan^{-1} \frac{x}{a}$	$\frac{a}{a^2 + x^2}$	$\frac{1}{a^2 + x^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a} + c$

## 2.2 Definite Integral

**Definition:**

$$\int_a^b f'(x) dx = [f(x)]_a^b = f(b) - f(a)$$

**Example 2.2.1** Find  $\int_1^9 (2 + \sqrt{x}) dx$

$$\begin{aligned} \text{Solution: } \int_1^9 (2 + \sqrt{x}) dx &= \int_1^9 (2 + x^{\frac{1}{2}}) dx \\ &= \left[ 2x + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_1^9 \\ &= \left[ 2x + \frac{2}{3}x^{\frac{3}{2}} \right]_1^9 \\ &= \left[ 2(9) + \frac{2}{3}(9)^{\frac{3}{2}} \right] - \left[ 2(1) + \frac{2}{3}(1) \right] \\ &= [18 + 18] - [2\frac{2}{3}] \\ &= 33\frac{1}{3} \end{aligned}$$

**Example 2.2.2** Find  $\int_1^3 (x^2 + 2e^{2x} + 1) dx$

$$\begin{aligned} \text{Solution: } \int_1^3 (x^2 + 2e^{2x} + 1) dx &= \left[ \frac{1}{3}x^3 + \frac{1}{2} \times 2e^{2x} + x \right]_1^3 \\ &= \left[ \frac{1}{3}(3)^3 + e^6 + 3 \right] - \left[ \frac{1}{3}(1)^3 + e^2 + 1 \right] \\ &= [9 + 403.43 + 3] - \left[ \frac{1}{3} + 7.39 + 1 \right] \\ &= 406.71 \end{aligned}$$

**Example 2.2.3** Find  $\int_1^2 \frac{1}{x^2} dx$

$$\begin{aligned} \text{Solution: } \int_1^2 \frac{1}{x^2} dx &= \int_1^2 x^{-2} dx \\ &= \left[ \frac{x^{(-2+1)}}{(-2+1)} \right]_1^2 \\ &= \left[ -\frac{1}{x} \right]_1^2 \\ &= \left( -\frac{1}{2} \right) - \left( -\frac{1}{1} \right) \\ &= \left( -\frac{1}{2} \right) + 1 = \frac{1}{2} \end{aligned}$$

**Exercise 2.2.1**

1.  $\int_1^2 e^{2x} dx$

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2.  $\int_{-1}^1 \frac{2}{x+2} dx$

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3.  $\int_0^1 (e^{3x} - 1) dx$

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4.  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x dx$

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### 2.3 Practical Exam Questions

#### Exercise 2.3.1

1.  $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$

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2.  $\int_1^{e^2} \frac{3}{x} \, dx$

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3.  $\int_0^{\ln 5} e^{-x} \, dx$

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4.  $\int_0^3 e^{3x} \, dx$

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**Exercise 2.3.2**

1.  $\int_0^1 e^{2x} dx$

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2.  $\int_1^3 \frac{1}{x+1} dx$

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3.  $\int_0^{\frac{\pi}{4}} \cos 2x dx$

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4.  $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos x dx$

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**2.4 Miscellaneous exercise****Exercise 2.4.1**

1.  $\int_{-1}^3 (2x^2 - 8x + 8) dx$

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2.  $\int_{-1}^{-\frac{1}{2}} (-x^3) dx$

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3.  $\int_0^1 (x^{\frac{1}{5}} - x^{\frac{1}{3}}) dx$

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4.  $\int_{\frac{1}{4}}^{\frac{1}{2}} \left( \frac{1}{x^2} - \frac{1}{x^3} \right) dx$

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