# 2/3 Unit Math Homework for Year 12

| | |
|---|---|
| **Student Name**: ____________ | **Grade**: ____________ |
| **Date**: ____________ | **Score**: ____________ |

## Table of contents



This edition was printed on June 23, 2014 with worked solutions.
Camera ready copy was prepared with the LaTeX2e typesetting system.

# 1 Tangent and the Derivative

## 1.1 Rules of Differentiation and Integration

| $f(x)$ | $f'(x)$ | $f(x)$ | $\int f(x).dx$ |
|---|---|---|---|
| $kx$ | $k$ | $k$ | $kx + c$ |
| $x^n$ | $nx^{n-1}$ | $x^n$ | $\frac{x^{n+1}}{n+1} + c$ |
| $ax^n + bx + c$ | $anx^{n-1} + b$ | $ax^n$ | $\frac{ax^{n+1}}{n+1} + c$ |
| $f\left(h(x)\right)$ | $f'\left(h(x)\right).h'(x)$ | $(ax+b)^n$ | $\frac{(ax+b)^{n+1}}{a(n+1)} + c$ |
| $g(x).h(x)$ | $g'(x).h(x) + g(x).h'(x)$ | $f'(x).(f(x))^n$ | $\frac{1}{n+1}(f(x))^{n+1} + c$ |
| $y = uv$ | $y' = uv' + u'v$ | | |
| $\frac{g(x)}{h(x)}$ | $\frac{g'(x).h(x) - g(x).h'(x)}{[h(x)]^2}$ | | |
| $y = \frac{u}{v}$ | $y' = \frac{u'v - uv'}{v^2}$ | | |
| $e^x$ | $e^x$ | $e^x$ | $= e^x + c$ |
| $e^{kx}$ | $ke^{kx}$ | $e^{kx}$ | $\frac{1}{k}e^{kx} + c$ |
| $e^{f(x)}$ | $f'(x)e^{f(x)}$ | $f'(x)e^{f(x)}$ | $e^{f(x)} + c$ |
| $\ln\, x$ | $\frac{1}{x}$ | $\frac{1}{x}$ | $\ln\, x + c$ |
| $\ln\, kx$ | $\frac{k}{x}$ | $\frac{k}{x}$ | $\frac{1}{k}\ln\, x + c$ |
| $\ln\, f(x)$ | $\frac{f'(x)}{f(x)}$ | $\frac{f'(x)}{f(x)}$ | $\ln\, f(x) + c$ |
| $\sin x$ | $\cos x$ | $\sin x$ | $-\cos x + c$ |
| $\sin kx$ | $k\cos kx$ | $\sin kx$ | $-\frac{1}{k}\cos kx + c$ |
| $\cos x$ | $-\sin x$ | $\cos x$ | $\sin x + c$ |
| $\cos kx$ | $-k\sin kx$ | $\cos kx$ | $\frac{1}{k}\sin kx + c$ |
| $\tan x$ | $\sec^2 x$ | $\sec^2 x$ | $\tan x + c$ |
| $\tan kx$ | $k\sec^2 kx$ | $\sec^2 kx$ | $\frac{1}{k}\tan kx + c$ |



## 1.2 Equation of Tangent and Normal

The derivative function is an expression for the gradient of the tangent to a curve, at any point.

**Example 1.2.1 Find the gradient of the tangent and normal to the curve $y = 2x^3$ at the point where x = 2.**

***Solution:*** $m_1 = \frac{d}{dx}(2x^3) = 6x^2 = 6 \times (2)^2 = 24$ *($m_1$ is gradient of tangent at $x = 2$) ;*
*gradient of normal* $m_2 = -\frac{1}{\text{gradient of tangent}} = -\frac{1}{24}$ *;*
*To find equation of tangent and normal, use $y - y_1 = m(x - x_1)$;*

## 1.3 Differentiation by the First Principles

**Definition:**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to c} \frac{f(x+c) - f(x)}{x - c}$$

**Example 1.3.1 Differentiate $f(x) = 3x^2$ from first principles.**

***Solution:***

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h} \\ &= \lim_{h\to 0} \frac{3(x^2 + 2hx + h^2) - 3x^2}{h} \\ &= \lim_{h\to 0} \frac{3x^2 + 6hx + 3h^2 - 3x^2}{h} \\ &= \lim_{h\to 0} \frac{3h(2x+h)}{h} = 6x \end{aligned}$$



**Exercise 1.3.1**

1. *Find* $\lim_{x\to 1} \frac{x^2+2x-3}{x-1}$

2. *Differentiate* $f(x) = x^2 + 4x + 5$ *from first principles.*

## 1.4 Differentiation of Power Functions

**Definition:**

$$\frac{d}{dx}(ax^n + bx + c) = anx^{n-1} + b$$

**Exercise 1.4.1**

1. $y = 2x^2 - 5x + 8$

2. $y = x^{\frac{1}{4}}$

3. $f(x) = \frac{1}{x^3} + \sqrt{x}$

4. $f(x) = 3x^4 + x\sqrt{x}$

5. $f(x) = 4\sin x + 3\cos x$

## 1.5 Chain Rule of Differentiation

**Definition:**

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

or if $y = f\left(h(x)\right)$ then

$$y' = f'\left(h(x)\right).h'(x)$$

**Exercise 1.5.1 Differentiate with respect to x:**

1. $\sqrt{x^2+4}$

2. $(2x+3)^6$

3. $sin\left(\frac{x}{2}\right)$

4. $sin^2\,x$

5. $e^{x^2+1}$

## 1.6 Product Rule of Differentiation

**Definition:** If $y = uv$, where u and v are functions of x, then

$$\frac{dy}{dx} = vu' + uv' = v\frac{du}{dx} + u\frac{dv}{dx}$$

or if $f(x) = g(x).h(x)$ then $f'(x) = g'(x).h(x) + g(x).h'(x)$

**Exercise 1.6.1 Differentiate with respect to x:**

1. $f(x) = x^4(1-x)^4$

2. $y = (2x+1)^5(2x-1)$

3. $f(x) = x\sqrt{1+x}$

4. $f(x) = \cos(log\, x)$

## 1.7 Quotient Rule of Differentiation

**Definition:** If $y = \frac{u}{v}$, where u and v are functions of x, then

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

or if $f(x) = \frac{g(x)}{h(x)}$ then $f'(x) = \frac{g'(x).h(x) - g(x).h'(x)}{[h(x)]^2}$

**Exercise 1.7.1 Differentiate with respect to x:**

1. $f(x) = \frac{3x-2}{2x+1}$

2. $f(x) = \frac{3x-4}{x^2+5}$

3. $y = \frac{x}{\cos x}$

4. $y = \frac{5+x}{5-x^2}$

## 1.8 Practical Exam Questions

**Exercise 1.8.1 Differentiate with respect to x:**

1. $f(x) = 3x^4 + x\sqrt{x}$

2. $f(x) = x^2 \log_e 2x$

3. $f(x) = (1 + sin\, x)^5$

4. $f(x) = x^2\, sin\, x$

5. $f(x) = (2e^x - 4)^8$

6. $f(x) = 5e^{2x}$

**Exercise 1.8.2 Differentiate with respect to x:**

1. $f(x) = cos(1 + x^3)$

2. $f(x) = log_e(1 + x^3)$

3. $f(x) = cos\,2x$

4. $f(x) = (x + 3)(x^3 - 3)$

5. $f(x) = sin\,2x + 3\,tan\,x$

6. $f(x) = log_e(1 + e^x)$