2/3 Unit Math Homework for Year 12

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

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13 Trigonometry 3

13.1 Integrals Involving Trigonometric Functions

Definition: $\int \cos ax\,dx = \frac{1}{a}\sin ax + c$ $\int \cos(ax+1)\,dx = \frac{1}{a}\sin ax + c.$
$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c.$ $\int \sin(ax+1)\,dx = -\frac{1}{a}\cos ax + c$
$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + c$ $\int \sec^2(ax+1)\,dx = \frac{1}{a}\tan ax + c.$

Example 13.1.1

1. *Find the primitive of* $\sin 5x$.

Solution: *From above,* $\int \sin 5x\,dx = -\frac{1}{5}\cos 5x + c$.

2. *Find* $\int sec^2 \frac{x}{5}\,dx$.

Solution: *From above,* $\int \sec^2 \frac{x}{5}\,dx = \frac{1}{\frac{1}{5}}\tan\frac{x}{5} + c = 5\tan\frac{x}{5} + c$

3. *Find the integral of* $\cos(3x+1)$.

Solution: *From above,* $\int \cos(3x+1)\,dx = \frac{1}{3}\sin(3x+1) + c$

Exercise 13.1.1 Find:

1. $\int \sin 4x\,dx$.

2. $\int \cos\frac{x}{4}\,dx$.

Indefinite integral (Primitive Function) :

$$\int (x^n + ax^n + b)\,dx = \frac{x^{n+1}}{n+1} + \frac{ax^{n+1}}{n+1} + bx + c$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,$$ where a, b, c and n are constants, $n \neq -1$

Definite Integral :

$$\int_a^b f'(x)\,dx = [f(x)]_a^b = f(b) - f(a)$$

Example 13.1.2

1. *Find* $\int_0^{\frac{\pi}{2}} \sin 2x\,dx$.

Solution:

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin 2x\,dx &= \left[-\tfrac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}} \\ &= \left(-\tfrac{1}{2}\cos 2\times\tfrac{\pi}{2}\right) - \left(-\tfrac{1}{2}\cos 2\times 0\right) \\ &= \left(-\tfrac{1}{2}\times(-1)\right) - \left(-\tfrac{1}{2}\times 1\right) \\ &= \tfrac{1}{2} + \tfrac{1}{2} = 1\end{aligned}$$

2. $\int_0^{\frac{\pi}{3}} \sin(2-x)\,dx$

Solution:

$$\begin{aligned}\int_0^{\frac{\pi}{3}} \sin(2-x)\,dx &= [(-1)\times -\cos(2-x)]_0^{\frac{\pi}{3}} \\ &= \cos(2-\tfrac{\pi}{3}) - \cos 2 \\ &= 0.9955\end{aligned}$$

3. $\int_{0.5}^1 \sec^2 x\,dx$

Solution:

$$\begin{aligned}\int_{0.5}^1 \sec^2 x\,dx &= [\tan x]_{0.5}^1 \\ &= \tan(1) - \tan(0.5) \\ &= 1.011\end{aligned}$$

Exercise 13.1.2 Find $\int_0^{\frac{\pi}{3}} \cos 3x\,dx$

Exercise 13.1.3 Find the integral of the following functions:

1. $\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \sec^2 2x\, dx$

2. *Find* $\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \cos 4x\, dx$

3. $\int_{0}^{\frac{\pi}{2}} \cos \frac{x}{2}\, dx$

4. $\int_{0}^{\frac{\pi}{4}} (sec^2 x - x)\, dx$

Exercise 13.1.4 Find $\int sec^2 4x\, dx$**.**

Exercise 13.1.5

1. *Differentiate* $y = \cos(3 - 2x)$.

2. *Hence find* $\int \sin(3 - 2x)$.

Exercise 13.1.6

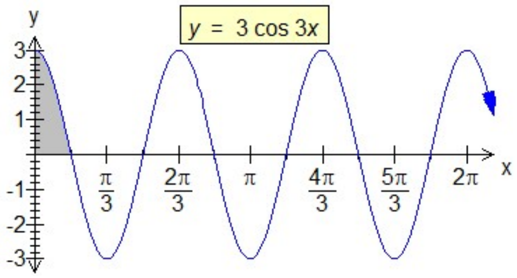
1. *Differentiate* $y = \cos^3 x$.

2. *Hence, evaluate* $\int_0^{\frac{\pi}{2}} \cos^2 x \sin x\, dx$

13.2 Applications of Integration

Example 13.2.1 Find the area under the curve $= 3\cos 3x$ **between** $x = 0$ **and** $x = \frac{\pi}{6}$**.**

Solution: *First, sketch the curve as shown below: (it's amplitude is 3 and period is* $\frac{2\pi}{3}$*.)*



Area: $A = \int_0^{\frac{\pi}{6}} 3\cos 3x\, dx = 3\left[\frac{1}{3}\sin 3x\right]_0^{\frac{\pi}{6}}$

$$= 3\left[\left(\frac{1}{3}\sin 3\times\frac{\pi}{3}\right) - \left(\frac{1}{3}\sin 3\times 0\right)\right]$$

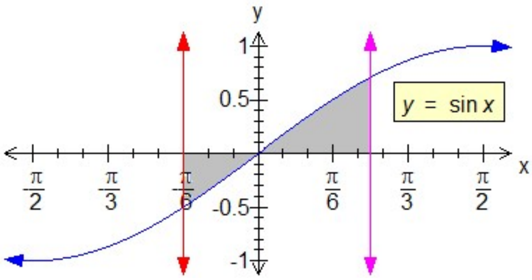
$$= 3\times\left(\frac{1}{3} - 0\right)$$

$$= 1\ units^2$$

Exercise 13.2.1 Find the area under the curve $y = 2\cos 3x$**, between** $x = -\frac{\pi}{6}$ **and** $x = \frac{\pi}{6}$**.**

Example 13.2.2 Find the area between the curve $y = \sin x$, **x-axis and the lines** $x = -\frac{\pi}{6}$ **and** $x = \frac{\pi}{4}$.

Solution: *First, sketch the curve as shown below:*



The parts of the area above and below the x axis must be found separately and then combined.

Area below the x axis: $A_1 = \left|\int_{-\frac{\pi}{6}}^{0} \sin x\, dx\right|$

$= \left|[-\cos x]_{-\frac{\pi}{6}}^{0}\right|$

$= |(-\cos 0) - (-\cos -(\frac{\pi}{6}))|$

$= |-1 + \frac{\sqrt{3}}{2}|$

$= 1 - \frac{\sqrt{3}}{2}\ units^2$

Area above the x-axis: $A_2 = \int_{0}^{\frac{\pi}{4}} \sin x\, dx = [-\cos x]_{0}^{\frac{\pi}{4}}$

$= (-\cos\frac{\pi}{4}) - (-\cos 0)$

$= -\frac{\sqrt{2}}{2} + 1\, units^2$

$\therefore$ *total area is* $A_1 + A_2 = 1 - \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} + 1 = 2 - \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\ units^2$

Exercise 13.2.2 Find the area enclosed by the curve $y = 2\sin x$, **the x-axis, and the lines** $x = -\frac{\pi}{3}$ **and** $x = \frac{\pi}{3}$

13.3 Practical Exam Questions

Exercise 13.3.1

1. *Find the area enclosed by the curve* $y = \cos 2x$*, the axis, and the lines* $x = -\frac{\pi}{6}$ *and* $x = \frac{\pi}{6}$*.*

2. *Find the area enclosed by the curve* $y = 3\sin x$*, the axis, and the lines* $x = -\frac{\pi}{6}$ *and* $x = \frac{\pi}{6}$*.*