# 2/3 Unit Math Homework for Year 12

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| **Student Name**: ______________ | **Grade**: ______________ |
| **Date**: ______________ | **Score**: ______________ |

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# 11 Trigonometry 1

## 11.1 The Fundamental Identity

**Definition:** $\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$
$1 + \cot^2\theta = cosec^2\theta$

**Example 11.1.1 Prove the following identities:**

1. $\frac{1-\sin^2\theta}{\sin^2\theta+\cos^2\theta} = \cos^2\theta$

**Solution:** $\frac{1-\sin^2\theta}{\sin^2\theta+\cos^2\theta} = \frac{\cos^2\theta}{1}$
$= cos^2\theta$

2. $\frac{\cos\theta}{1-\sin\theta} - \tan\theta = \sec\theta$

**Solution:** $\frac{\cos\theta}{1-\sin\theta} - \tan\theta = \frac{\cos\theta}{1-\sin\theta} - \frac{\sin\theta}{\cos\theta}$
$= \frac{\cos^2\theta - \sin\theta(1-\sin\theta)}{(1-\sin\theta)\cos\theta}$
$= \frac{1-\sin\theta}{(1-\sin\theta)\cos\theta}$
$= \frac{1}{\cos\theta}$
$= \sec\theta$

3. $\sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta = \sin^2\alpha - \sin^2\beta$

**Solution:** $\sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta = \sin^2\alpha(1-\sin^2\beta) - (1-\sin^2\alpha)\sin^2\beta$
$= \sin^2\alpha - \sin^2\alpha\sin^2\beta - (\sin^2\beta - \sin^2\alpha\sin^2\beta)$
$= \sin^2\alpha - \sin^2\beta$

4. $\sec\theta + \tan\theta = \frac{1+\sin\theta}{\cos\theta}$

**Solution:** $\sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$
$= \frac{1+\sin\theta}{\cos\theta}$

## Exercise 11.1.1

1. $\frac{\cos\theta\cot\theta}{\cot\theta+\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$

2. $\frac{1+\cot\theta}{cosec\,\theta} - \frac{\sec\theta}{\tan\theta+\cot\theta} = \cos\theta$

3. $\frac{1-\cos^2\theta}{\cos\theta} = \sin\theta\tan\theta$

4. $1 + \frac{1}{\cot^2\theta} = \frac{1}{\cos^2\theta}$

5. $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

## 11.2 Symmetry properties of trigonometric Functions

Symmetry properties

| Degrees | Radians |
|---|---|
| $\sin(90^\circ - \theta) = \cos\theta$<br>$\cos(90^\circ - \theta) = \sin\theta$<br>$\tan(90^\circ - \theta) = \cot\theta$ | $\sin(\frac{\pi}{2} - \theta) = \cos\theta$<br>$\cos(\frac{\pi}{2} - \theta) = \sin\theta$<br>$\tan(\frac{\pi}{2} - \theta) = \cot\theta$ |
| $\sin(180^\circ - \theta) = \sin\theta$<br>$\cos(180^\circ - \theta) = -\cos\theta$<br>$\tan(180^\circ - \theta) = -\tan\theta$ | $\sin(\pi - \theta) = \sin\theta$<br>$\cos(\pi - \theta) = -\cos\theta$<br>$\tan(\pi - \theta) = -\tan\theta$ |
| $\sin(180^\circ + \theta) = -\sin\theta$<br>$\cos(180^\circ + \theta) = -\cos\theta$<br>$\tan(180^\circ + \theta) = \tan\theta$ | $\sin(\pi + \theta) = -\sin\theta$<br>$\cos(\pi + \theta) = -\cos\theta$<br>$\tan(\pi + \theta) = \tan\theta$ |
| $\sin(360^\circ - \theta) = -\sin\theta$<br>$\cos(360^\circ - \theta) = \cos\theta$<br>$\tan(360^\circ - \theta) = -\tan\theta$ | $\sin(2\pi - \theta) = -\sin\theta$<br>$\cos(2\pi - \theta) = \cos\theta$<br>$\tan(2\pi - \theta) = -\tan\theta$ |
| $\sin(360^\circ + \theta) = \sin\theta$<br>$\cos(360^\circ + \theta) = \cos\theta$<br>$\tan(360^\circ + \theta) = \tan\theta$ | $\sin(2\pi + \theta) = \sin\theta$<br>$\cos(2\pi + \theta) = \cos\theta$<br>$\tan(2\pi + \theta) = \tan\theta$ |

Some exact values

| | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ |
|---|---|---|---|---|---|
| sin | $0$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $1$ |
| cos | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $0$ |
| tan | $0$ | $\frac{1}{\sqrt{3}}$ | $\sqrt{3}$ | $1$ | $-$ |
| *cosec* | $-$ | $2$ | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | $1$ |
| sec | $1$ | $\frac{2}{\sqrt{3}}$ | $2$ | $\sqrt{2}$ | $-$ |
| cot | $-$ | $\sqrt{3}$ | $\frac{1}{\sqrt{3}}$ | $1$ | $0$ |

**Example 11.2.1 Find all values of $\theta$ between 0 and $2\pi$ for which:**

*1.* $\sin\theta = -\frac{1}{\sqrt{2}}$

***Solution:*** $\sin\theta = -\frac{1}{\sqrt{2}}$, *Since $\sin\theta$ is negative, $\theta$ lies in the 3rd and 4th quadrants.*

$\therefore \theta = \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$= \frac{5\pi}{4}, \frac{7\pi}{4}$

*2.* $\cos\theta = \frac{\sqrt{3}}{2}$

***Solution:*** $\cos\theta = \frac{\sqrt{3}}{2}$, *Since $\cos\theta$ is positive, $\theta$ lies in the 1st and 4th quadrants.*

$\therefore \theta = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{11\pi}{6}$

**Exercise 11.2.1**

*1.* $\sec\theta = -\frac{2}{\sqrt{3}}$

*2.* $\sin\theta = \frac{1}{2}$

*3.* $\cos\theta = -\frac{1}{2}$

*4.* $\sin\theta = -1$

*5.* $\tan\theta = -\sqrt{3}$

## 11.3 Derivative of $\sin x$, $\cos x$ and $\tan x$

**Definition:** $\frac{d}{dx}(\sin x) = \cos x$
$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan x) = \sec^2 x$

## 11.4 Derivative of $\sin(ax+b)$, $\cos(ax+b)$ and $\tan(ax+b)$

**Definition:** $\frac{d}{dx}\sin(ax+b) = a\cos(ax+b)$
$\frac{d}{dx}\cos(ax+b) = -a\sin(ax+b)$
$\frac{d}{dx}\tan(ax+b) = a\sec^2(ax+b)$

**Example 11.4.1 Differentiate the following:**

1. $\sin 5x$.

***Solution:*** *let* $y = \sin 5x$, $\frac{dy}{dx} = 5\cos 5x$

2. $\cos^2 x$.

***Solution:*** *Let* $y = \cos^2 x = u^2$, *where* $u = \cos x$
$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 2u \times -\sin x = -2\cos x \sin x$

3. $\sin(x^2)$.

***Solution:*** *Let* $y = \sin(x^2) = \sin u$, *where* $u = x^2$,
$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times 2x = 2x\cos(x^2)$

4. $x^2 \sin x$.

***Solution:*** *Let* $y = x^2 \sin x = uv$, *where* $u = x^2$ *and* $v = \sin x$

$$\begin{aligned} \frac{dy}{dx} &= v.\frac{du}{dx} + u.\frac{dv}{dx} \quad \textit{(product rule)} \\ &= \sin x.2x + x^2\cos x \\ &= 2x\sin x + x^2\cos x \end{aligned}$$

**Exercise 11.4.1 Differentiate the following with respect to x:**

1. $x^2 \cos 2x$

2. $x(3\cos x + 2\sin x)$.

3. $\cos^2 3x$

4. $\tan(x^2 - 1)$

5. $\frac{1}{2}\cos 2x + x\sin x$

## 11.5 Practical Exam Questions

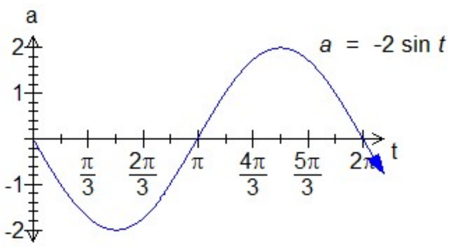
**Exercise 11.5.1 A particle P is moving along the x axis. Its position at time t seconds is given by** $x = 2\sin t - t$**, where** $t \leq 0$**.**

1. *Find an expression for velocity of the particle.*

2. *In what direction is the particle moving at t = 0?*

3. *Determine when the particle first comes to rest.*

4. *When is the acceleration negative for* $0 \leq t \leq 2\pi$*?*



5. *Calculate the total distance travelled by the particle in the first* $\pi$ *seconds.*