4 Unit Math Homework for Year 12

Student Name: ______________	**Grade**: ______________
Date: ______________	**Score**: ______________

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This edition was printed on June 16, 2017 with worked solutions.

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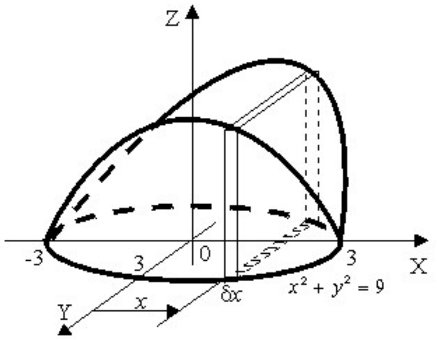
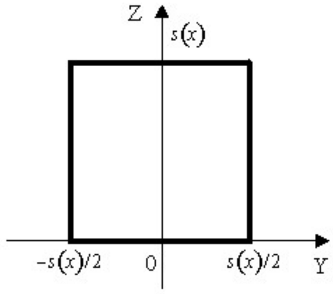
5 Topic 5 — Volumes Part 3

5.4 Volumes of solid with parallel cross-sections of similar shapes

Example 5.4.1

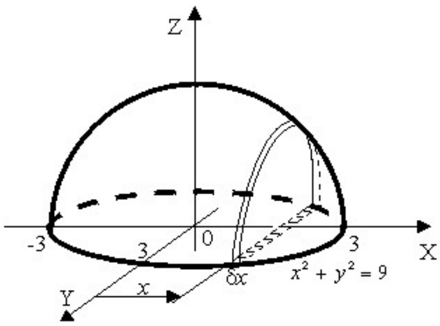
1. *The base of a particular solid is the circle $x^2 + y^2 = 9$. Every cross-section perpendicular to the x- axis is a square with one side in the base of the solid. Find the volume of the solid.*

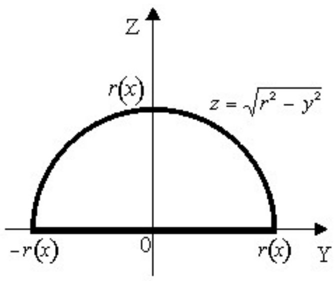




Solution: *The slice is a square with area of cross-section A, thickness δx.*
$A(x) = s^2(x)$, *where* $s(x) = 2\sqrt{9 - x^2} \Rightarrow \therefore A(x) = 4(x - 9^2)$
The slice has volume $\delta V = A(x)\delta x = 4(9 - x^2)\,\delta x$.
$\therefore\ V = \lim_{\delta x \to 0} \sum_{x=-3}^{3} 4(9 - x^2)\,\delta x$
$= 4\int_{-3}^{3} (9 - x^2)\,dx$
$= \left[\left(9x - \frac{x^3}{3}\right)\right]_{-3}^{3}$
$= 144\ units^3$

2. *The base of a particular solid is the circle . Every cross-section perpendicular to the x- axis is a semicircle with diameter in the base of the solid. Find the volume of the solid.*



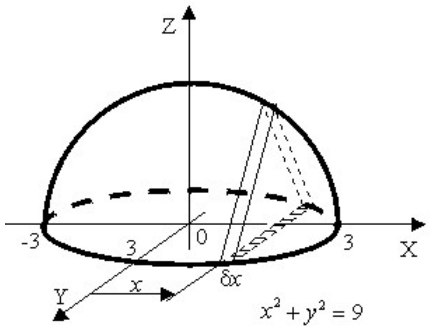


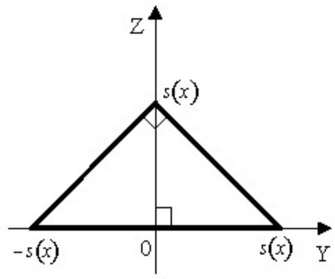
Solution: *The slice is a semicircle with area of cross-section A, thickness δx.*
$A(x) = \frac{\pi r^2(x)}{2}$ *where* $r(x) = \sqrt{9 - x^2} \Rightarrow A(x) = \frac{\pi(9-x^2)}{2}$.
The slice has volume $\delta V = A(x)\delta x = \frac{\pi(9-x^2)}{2}\,\delta x$
$\therefore\ V = \lim_{\delta x \to 0} \sum_{x=-3}^{3} \frac{\pi(9-x^2)}{2}\,\delta x$

$= \frac{\pi}{2} \int_{-3}^{3} (9 - x^2)\, dx$
$= \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$
$= 18\pi \ units^3.$

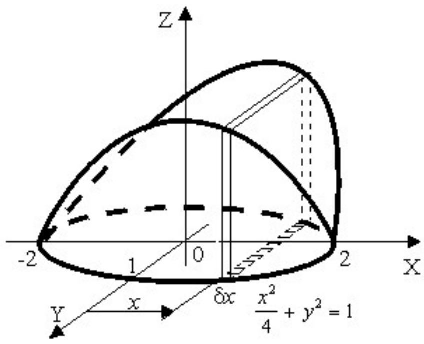
Exercise 5.4.1

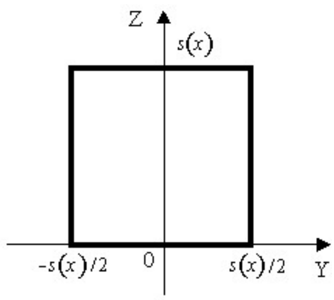
1. *The base of a particular solid is the circle* $x^2 + y^2 = 9$ *Every cross-section perpendicular to the x- axis is an isosceles right-angled triangle with hypotenuse in the base of the solid. Find the volume of the solid.*





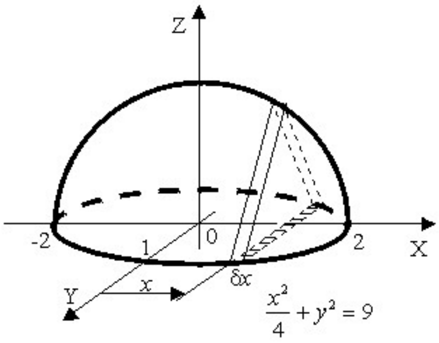
2. *The base of a particular solid is the ellipse* $\frac{x^2}{4} + y^2 = 1$. *Every cross-section perpendicular to the major axis is a square with one side in the base of the solid. Find the volume of the solid.*

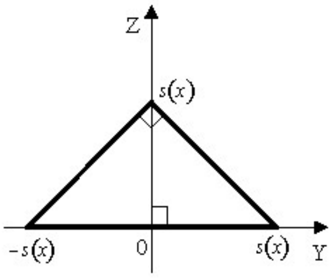




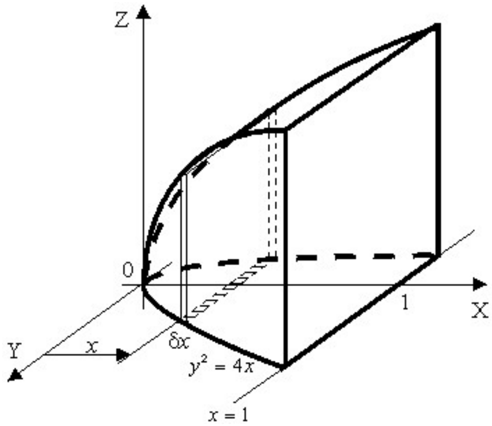
Exercise 5.4.2

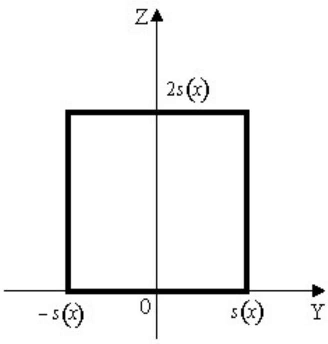
1. *The base of a particular solid is the ellipse* $\frac{x^2}{4} + y^2 = 1$ *Every cross-section perpendicular to the major axis is an isosceles right-angled triangle with hypotenuse in the base of the solid.*





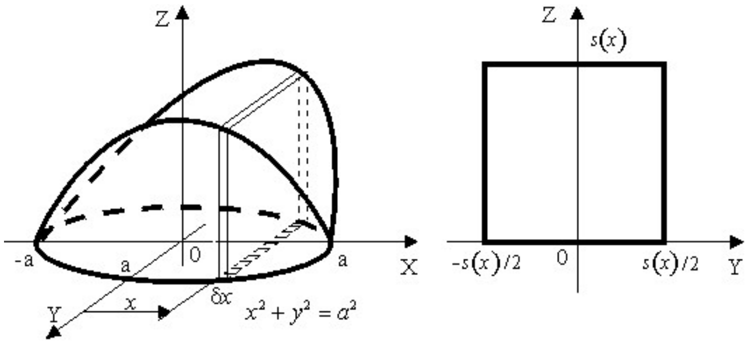
2. *The base of a particular solid is the region bounded by the parabola* $y^2 = 4x$ *between its vertex and its latus rectum. Every cross-section perpendicular to the x- axis is a square with one side in the base of the solid. Find the volume of the solid.*
Hint: The latus rectum of the parabola $y^2 = 4x$ *is the line* $x = 1$.



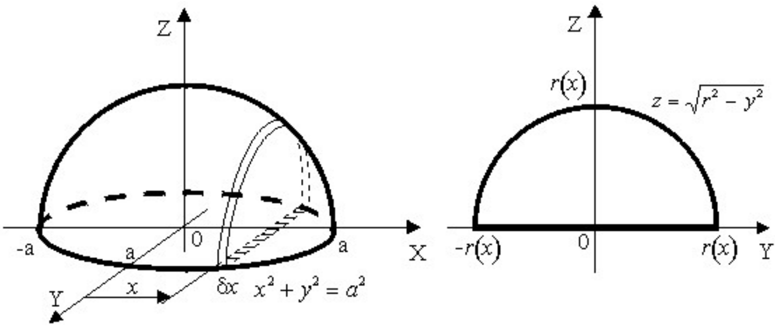


Exercise 5.4.3

1. *The base of a particular solid is the circle* $x^2 + y^2 = a^2$ *Every cross-section perpendicular to the x- axis is a square with one side in the base of the solid. Find the volume of the solid.*

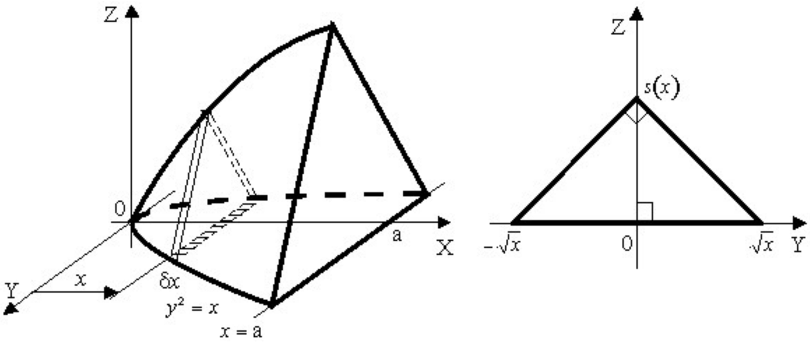


2. *The base of a particular solid is the circle* $x^2 + y^2 = a^2$ *Every cross-section perpendicular to the x- axis is a semicircle with diameter in the base of the solid. Find the volume of the solid.*

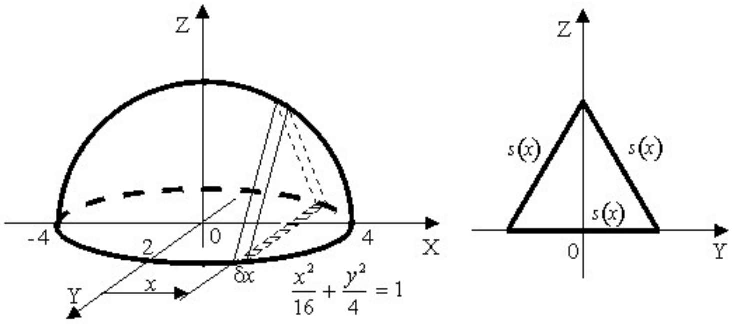


Exercise 5.4.4

1. *The base of a solid is a segment of the parabola $y^2 = x$ cut of by the line $x = a$. Cross-sections taken perpendicular to the axis of the parabola are right-angled isosceles triangles with hypotenuse in the base of the solid. Find the volume of the solid.*

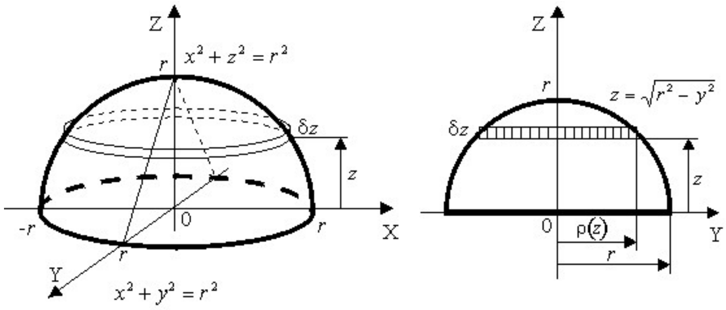


2. *The base of a particular solid is the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$. Every cross-section perpendicular to the major axis of the ellipse is an equilateral triangle with one side in the base of the solid. Find the volume of the solid.*

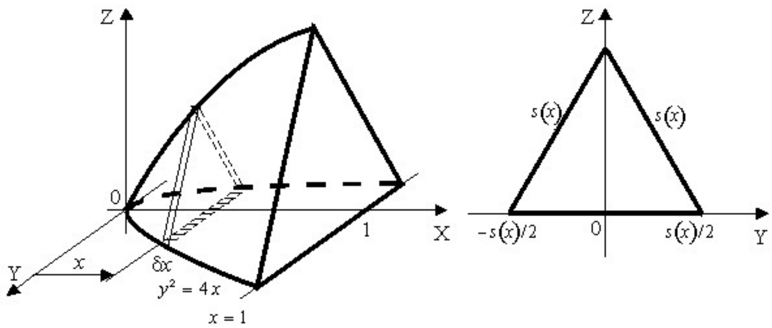


Exercise 5.4.5

1. *A hemisphere has radius r. By considering cross-sections parallel to the base of the hemisphere, show that its volume is given by $V = \frac{2}{3}\pi r^3$.*

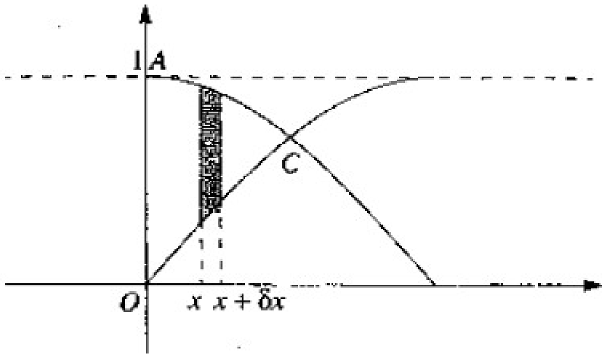


2. *The base of a particular solid is the region bounded by the parabola y=$4x$ between its vertex $(0,\ 0)$ and its latus rectum . Every cross-section perpendicular to the x- axis is an equilateral triangle with one side in the base of the solid. Find the volume of the solid.*
Hint: The latus rectum of the parabola $y^2 = 4x$ is the line $x = 1$.



5.5 Past Exam Questions

Exercise 5.5.1 The digram below shoes part of the graphs of $y = \cos x$ and $y = \sin x$. The graph of $y = \cos x$ meets the y-axis at A, and the C is the first point of intersection of the two graphs to the right o the y-axis. The region OAC is to be rotated about the line $y = 1$.



1. *Write down the coordinates of the point C.*

2. *The shaded strip of width δx shown in the diagram is rotated about the line $y = 1$.*
   Show that the volume δV of the resulting slice is given by:
   $$\delta V = \pi(2\cos x - 2\sin x + \sin^2 x - \cos^2 x)\,\delta x.$$

3. *Hence evaluate the total volume when the region OAC is rotated about the line $y = 1$.*