

4 Unit Math Homework for Year 12

Student Name: _____	Grade: _____
Date: _____	Score: _____

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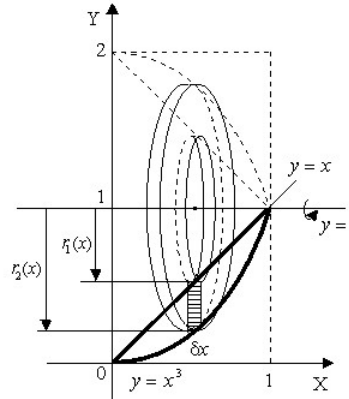
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5 Topic 5 — Volumes Part 2

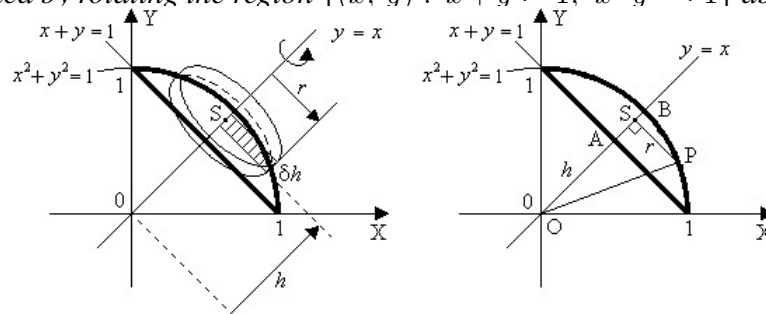
5.2 Volumes of solids of revolution by slicing

Exercise 5.2.1

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, x^3 \leq y \leq x\}$ about the line $y = 1$.



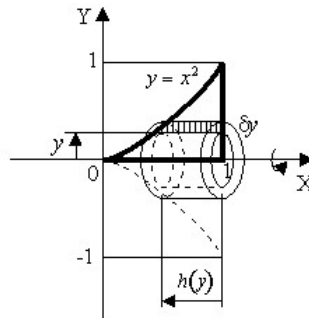
- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : x + y > 1, x^2 + y^2 < 1\}$ about the line $y = x$.



5.3 Volumes of solids of revolution by cylindrical shells

Example 5.3.1

1. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$ about the x -axis.



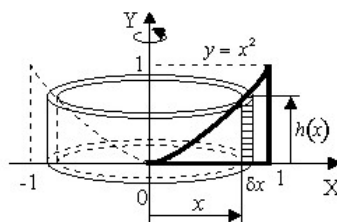
Solution: The typical cylindrical shell has radii $y, y + \delta y$, and height $h(y) = 1 - \sqrt{y}$.

This shell has volume: $\delta V = \pi[(y + \delta y)^2 - y^2] \times h(y)$

$$= 2\pi(1 - \sqrt{y})y \delta y \text{ (ignoring } (\delta y)^2).$$

$$\begin{aligned} \therefore V &= \lim_{\delta y \rightarrow 0} \sum_{y=0}^1 2\pi(1 - \sqrt{y})y \delta y \\ &= 2\pi \int_0^1 (1 - \sqrt{y})y \, dy \\ &= 2\pi \left[\left(\frac{y^2}{2} - \frac{y^{\frac{5}{2}}}{\frac{5}{2}} \right) \right]_0^1 \\ &= \frac{\pi}{5} \text{ units}^3. \end{aligned}$$

2. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$ about the y -axis.



Solution: The typical cylindrical shell has radii $x, x + \delta x$, and height $h(x) = x^2$.

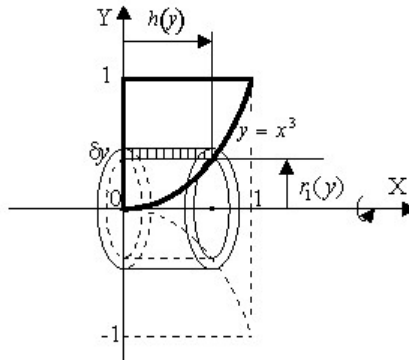
This shell has volume: $\delta V = \pi[(x + \delta x)^2 - x^2] \times h(x)$

$$= 2\pi x^3 \delta x \text{ (ignoring } (\delta x)^2).$$

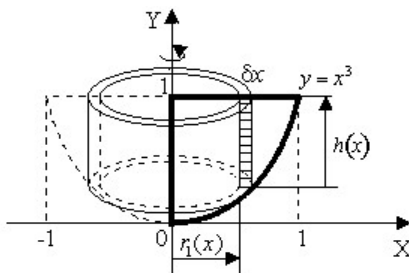
$$\begin{aligned} \therefore V &= \lim_{\delta x \rightarrow 0} \sum_{x=0}^1 2\pi x^3 \delta x \\ &= 2\pi \int_0^1 x^3 \, dx \\ &= 2\pi \left[\frac{x^4}{4} \right]_0^1 \\ &= \frac{\pi}{2} \text{ units}^3. \end{aligned}$$

Exercise 5.3.1

- By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, x^3 \leq y \leq 1\}$ about the x -axis.

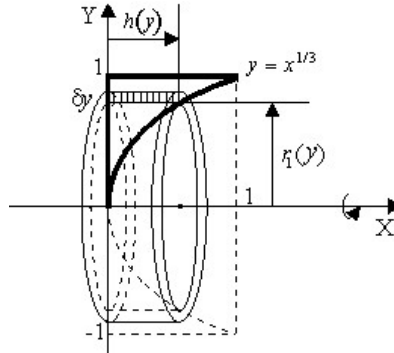


- By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, x^3 \leq y \leq 1\}$ about the y -axis.

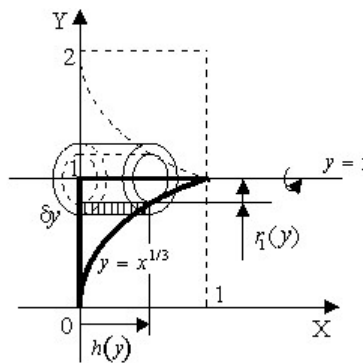


Exercise 5.3.2

- By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, x^{\frac{1}{3}} \leq y \leq 1\}$ about the x -axis.

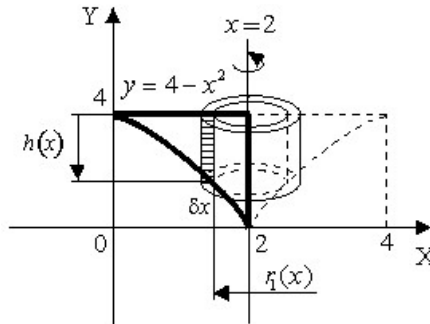


- By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, x^{\frac{1}{3}} \leq y \leq 1\}$ about the line $y = 1$.

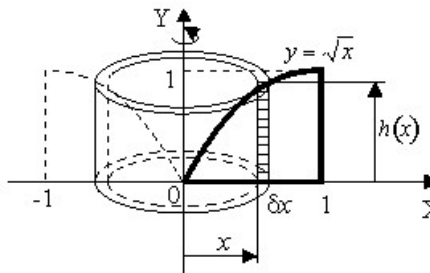


Exercise 5.3.3

- By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2, 4 - x^2 \leq y \leq 4\}$ about the line $x = 2$.

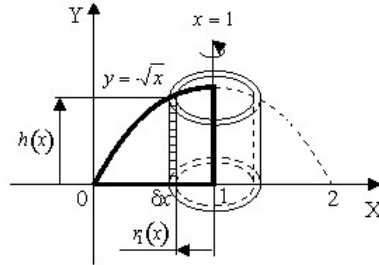


- By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x}\}$ about the y -axis.

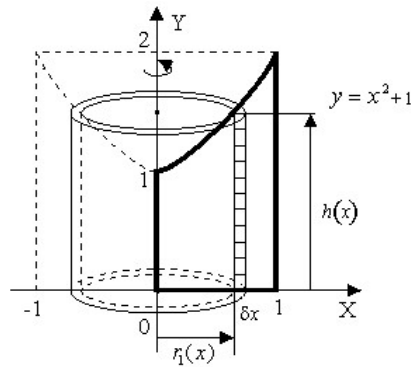


Exercise 5.3.4

- By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x}\}$ about the line $x = 1$.

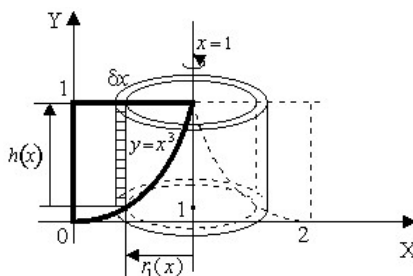


- By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2 + 1\}$ about the y-axis.



Exercise 5.3.5

1. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq x, x^3 \leq y \leq 1\}$ about the line $x = 1$.



2. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq x, x^3 \leq y \leq 1\}$ about the line $y = 1$.

