

4 Unit Math Homework for Year 12

Student Name: _____	Grade: _____
Date: _____	Score: _____

Table of contents

5	Topic 5 — Volumes Part 1	1
5.1	Volumes of solids of revolution by slicing	1

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5 Topic 5 — Volumes Part 1

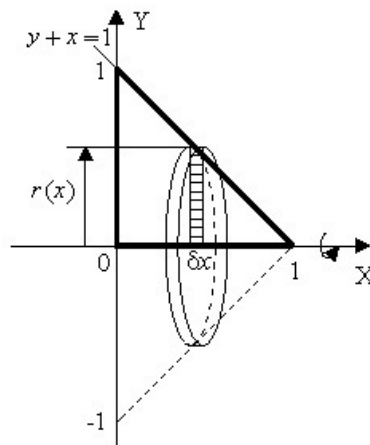
5.1 Volumes of solids of revolution by slicing

Definite integrals as limiting sums:

$$A = \int_a^b f(x) dx, \Rightarrow \therefore \lim_{\delta x \rightarrow 0} \sum_{x=a}^b f(x) \delta x = \int_a^b f(x) dx$$

Volumes of solids of revolution: A solid of revolution is formed when a bounded region of the Cartesian plane is rotated about a fixed line in the plane called the axis of rotation.

Example 5.1.1 By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : x \geq 0, y \geq 0, y + x \leq 1\}$ about the x-axis.



Solution: A slice perpendicular to axis of rotation is a disc of thickness δx and radius $r = y$.

But $y + x = 1$, hence $r(x) = 1 - x$.

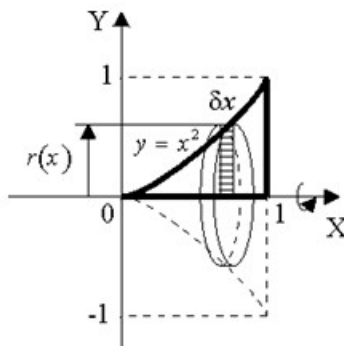
The slice has volume $\delta V = \pi r^2 \delta x \Rightarrow \delta V = \pi(1 - x)^2 \delta x$.

$$\begin{aligned} \therefore V &= \lim_{\delta x \rightarrow 0} \sum_{x=0}^1 \pi(1 - x)^2 \delta x \\ &= \int_0^1 \pi(1 - x)^2 dx \\ &= \left[\frac{\pi(1-x)^3}{3} \right]_0^1 \\ &= \frac{\pi}{3}. \end{aligned}$$

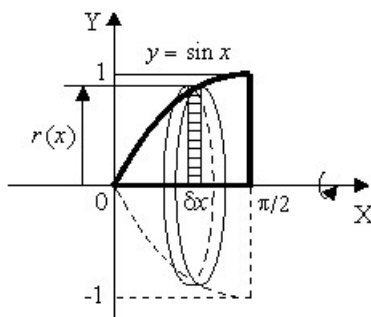
\therefore the volume of the solid is $\frac{\pi}{3}$ cubic units.

Exercise 5.1.1

1. By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$ about the x -axis.

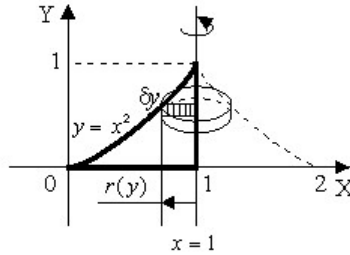


2. By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \sin x\}$ about the x -axis.

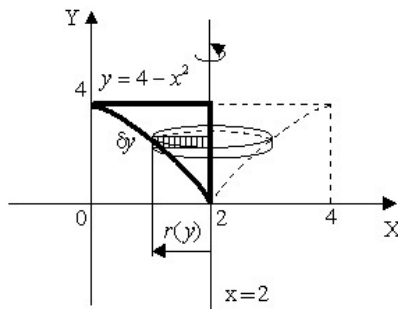


Exercise 5.1.2

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$ about the line $x = 1$.

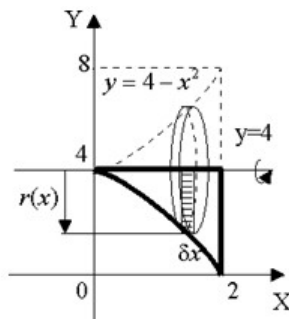


- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2, 4 - x^2 \leq y \leq 4\}$ about the line $x = 2$.

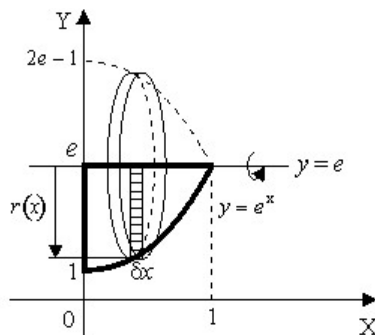


Exercise 5.1.3

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2, 4 - x^2 \leq y \leq 4\}$ about the line $y = 4$.

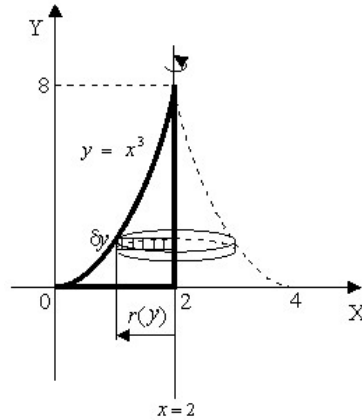


- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, e^x \leq y \leq e\}$ about the line $y = e$.

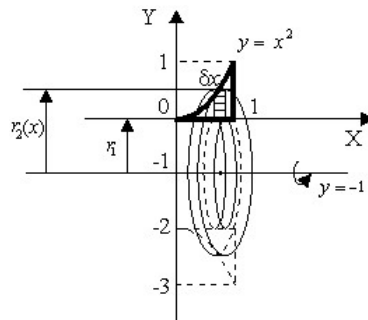


Exercise 5.1.4

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq x^3\}$ about the line $x = 2$.

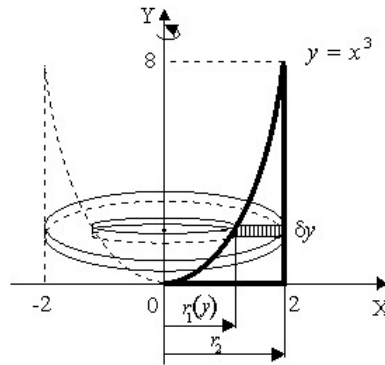


- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$ about the line $y = -1$.

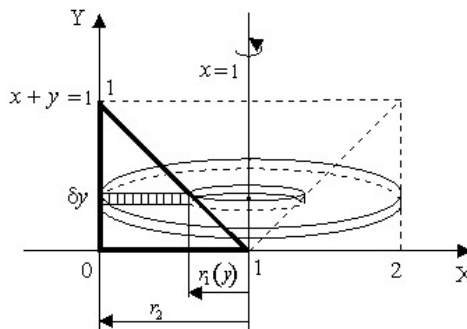


Exercise 5.1.5

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq x^3\}$ about the y -axis.

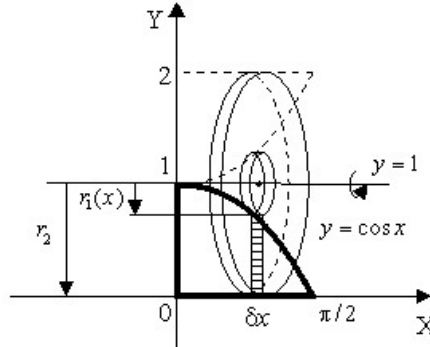


- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : x \geq 0, y \geq 0, y + x \leq 1\}$ about the line $x = 1$.

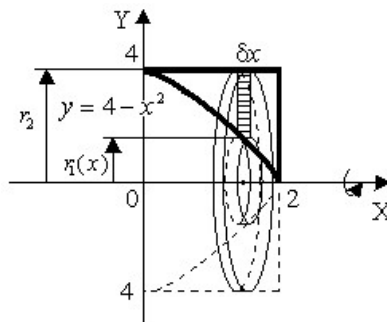


Exercise 5.1.6

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \cos x\}$ about the line $y = 1$.

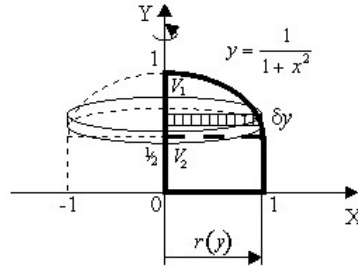


- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2, 4 - x^2 \leq y \leq 4\}$ about the x-axis.

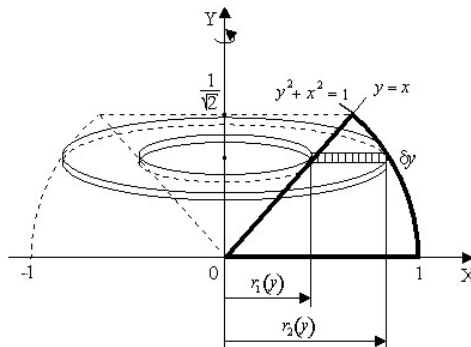


Exercise 5.1.7

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq \frac{1}{1+x^2}\}$ about the y-axis.

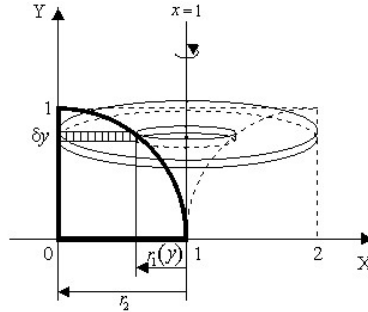


- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : x \geq 0, 0 \leq y \leq x, x^2 + y^2 \leq 1\}$ about the y-axis.

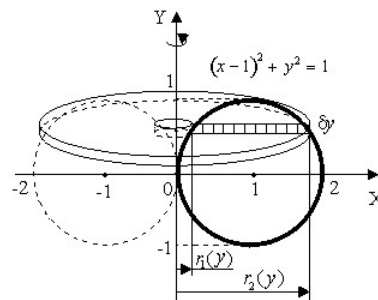


Exercise 5.1.8

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$ about the line $x = 1$.

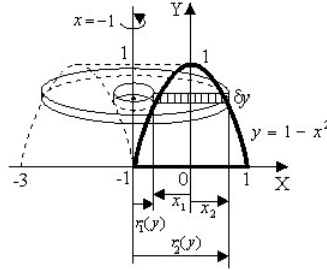


- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region enclosed within the circle $(x - 1)^2 + y^2 = 1$ about the y -axis.

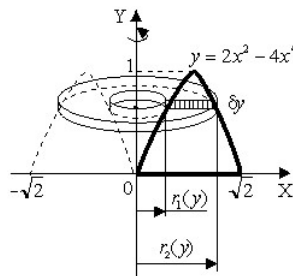


Exercise 5.1.9

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq 1 - x^2\}$ about the line $x = -1$.

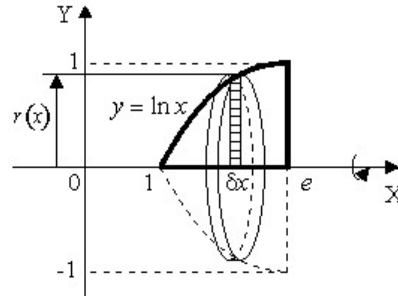


- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq \sqrt{2}, 0 \leq y \leq 2x^2 - 4x^4\}$ about the y -axis.



Exercise 5.1.10

- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq e, 0 \leq y \leq \ln x\}$ about the x -axis.



- By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq x^3\}$ about the line $y = 8$.

