4 Unit Math Homework for Year 12

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

Table of contents

This edition was printed on June 16, 2017 with worked solutions.

Camera ready copy was prepared with the LaTeX2e typesetting system.

7 Topic 7 — Mechanics Part 3

7.5 The Conical Pendulum

Example 7.5.1

1. *A bob P of mass m is suspended from a fixed point A by a light inextensible string of length ℓ. P is observed to perform uniform circular motion with angular velocity ω in a plane below A. Show that the depth of the bob below the suspension point is independent of the length of he string. Describe what happens if the rotational speed of the bob is increased.*

Solution: *The resultant force is $mr\omega^2$ towards the centre of the circle.*
The resultant has a vertical component zero $\Rightarrow$ $T\cos\theta = mg$.
The resultant has a horizontal component $mr\omega^2$ $\Rightarrow$ $T\sin\theta = m(\ell\sin\theta)\omega^2$.

$$\begin{cases} T\cos\theta = mg & (1) \\ T = m\ell\omega^2 & (2) \end{cases} \Rightarrow \quad \begin{array}{ll} \cos\theta = \frac{g}{\ell\omega^2} & (1)\div(2) \\ \therefore h = \frac{g}{\omega^2} & \end{array}$$

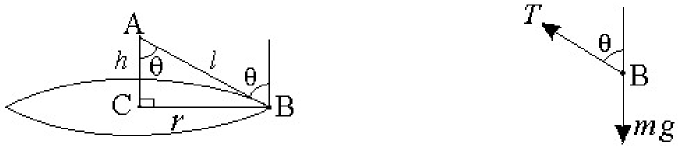
Hence h is independent of ℓ and as ω increase, h decreases and the bob rises.
The system of string and bob described in this example is called a conical pendulum.

2. *A string passing through a smooth hole in a smooth horizontal table connects a particle P of mass m on the table to a particle Q of mass m suspended below the table. P and Q are both performing uniform circular motion with angular velocity ω, where Q moves in a horizontal circle at a depth h below the table. the lengths of string above and below the table are ℓ and L respectively. Show that h depends only on ω, and that $\frac{\ell}{L} = \frac{M}{m}$.*

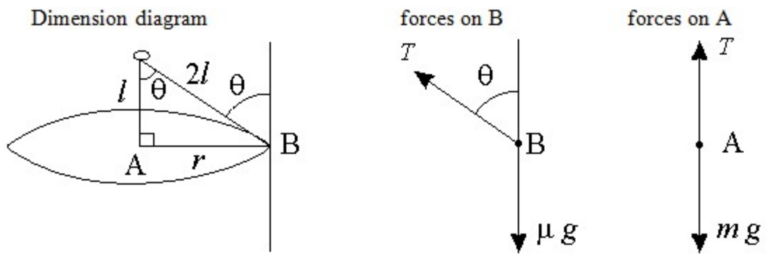
Solution: *The resultant force on P is horizontal, with magnitude $m\ell\omega^2$.*
The resultant force On Q is horizontal, with magnitude $Mr\omega^2 = M(L\sin\theta)\omega^2$.

Exercise 7.5.1

1. *An inextensible string of length $2\,m$ is fixed at one end A and carries at its other end B a particle of mass $6\,kg$ which is rotating in a horizontal circle whose center is $1\,m$ vertically below A. Find the tension in the string and the angular velocity of the particle.*



2. *A light inextensible string of length 3ℓ is threaded through a smooth ring and carries a particle at each end. One particle A, of mass m, is at rest at a distance ℓ below the ring while the other particle B, of mass μ, is rotating in a horizontal circle whose center is A. Find:*

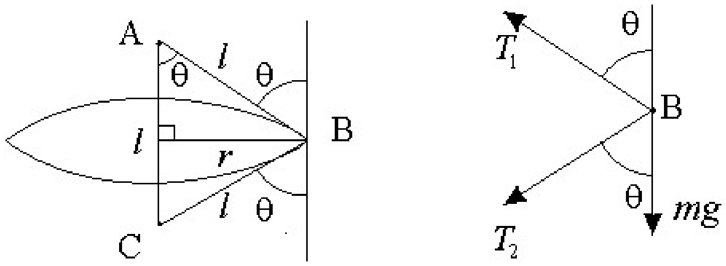


   (a) *m in terms of μ.*

   (b) *The angular velocity of B*

Exercise 7.5.2

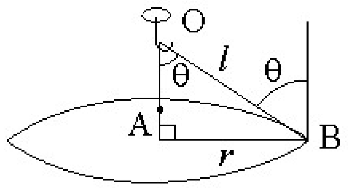
1. *Two light inextensible strings AB and BC each of length ℓ are attached to a particle of mass m at B. The other ends A and C are fixed to two points in a vertical line such that A is a distance ℓ above C. The particle describes a horizontal circle with constant angular velocity . Find:*

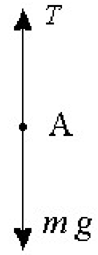


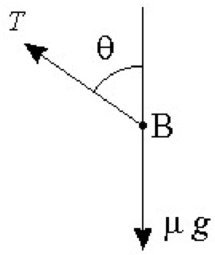
   (a) *the tension in the strings*

   (b) *the least value of in order that strings are taut.*

2. *Two particles A and Forces on engine of masses m and μ respectively are attached to the ends of a light inextensible string which passes over a smooth hook at O which is free to rotate. The particle A hangs at rest vertically below O while the particle B moves in a horizontal circle with constant speed v. Find expressions for:*



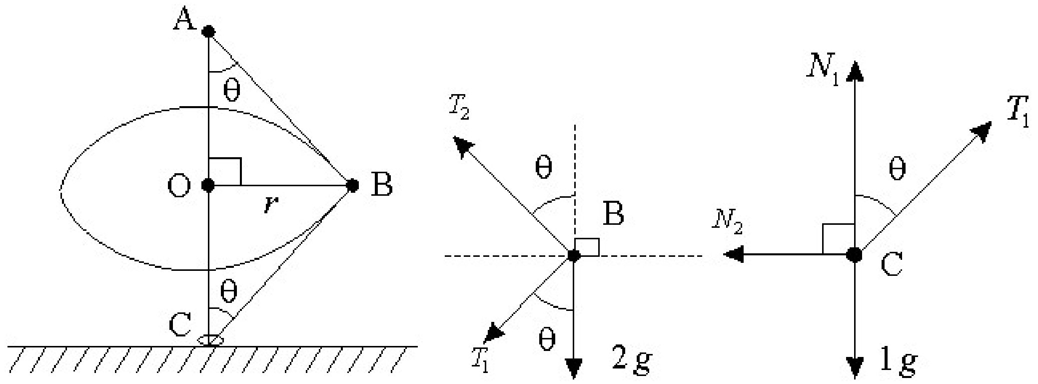




   (a) *the length OB and the angle AOB,*

   (b) *the radius of the circle in which B moves.*

Exercise 7.5.3 Two rigid light rods AB and BC, each of length $0.5\,m$, are smoothly jointed at B and the rod AB is smoothly jointed at A to a fixed smooth vertical rod. The joint at B has a particle of mass $2\,kg$ attached. A small ring of mass $1\,kg$ is smoothly jointed to BC at C and can slide on the vertical rod below A. The ring rests on a smooth horizontal ledge at a distance $\frac{\sqrt{3}}{2}\,m$ below A. The system rotates about the vertical rod with constant angular velocity 6 radians per second. Find :



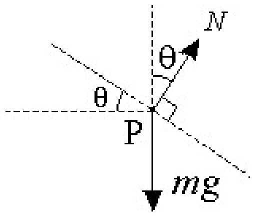
1. *the forces in the rod AB and BC,*

2. *the force exerted by the ledge on the ring.*

7.6 Motion Around a Circular Banked Track

Exercise 7.6.1

1. *A car has no tendency to slip when travelling at a speed of $v\,ms^{-1}$ round a section of track of radius $100\,m$ which is banked at an angle of $12°$. Taking $g = 9.80\,ms^{-2}$, find the speed of the car.*

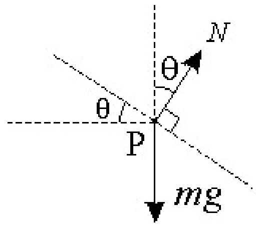


2. *A car has no tendency to slip when travelling at a speed of $30\,ms^{-1}$ round a section of track of radius $200\,m$ which is banked at an angle of $\theta°$. Find the angle of banking of the track, taking $g = 9.80\,ms^{-2}$.*

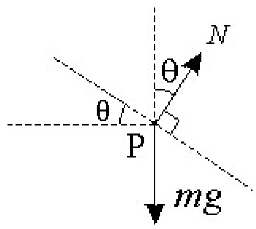
3. *An aircraft is flying at a speed of $100\,ms^{-1}$ in a horizontal circle of radius $4\,km$. Taking $g = 9.80\,ms^{-2}$, find at what angle the aircraft is banked.*

Exercise 7.6.2

1. *At what speed should a car travel round a bend of radius* $80\,m$ *which is banked at an angle of* 10°.

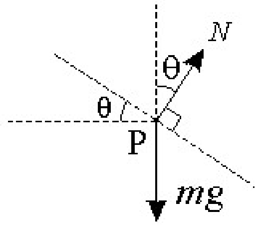


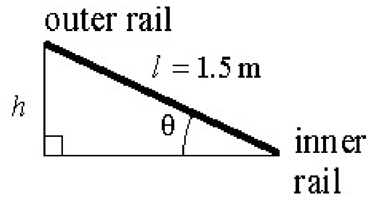
2. *A bend on a racetrack is designed with variable banking so that cars on the inside can corner at* $80\,kmh^{-1}$ *and those on the outside can corner at* $160\,kmh^{-1}$*, in both cases without any tendency to slip. If the inner radius is* $200\,m$ *and the outer radius is* $220\,m$*, find the difference between the angles of banking at the inside and the outside of the track.*



Exercise 7.6.3

1. *A railway line is taken round a circular bend of radius* $1000\,m$*. The distance between the rails is* $1.5\,m$*. At what height above the inner rail should the outer rail be raised in order to eliminate lateral thrust for an engine travelling at a speed of* $40\,km/s$ *round the bend?*





2. *A railway line has been constructed around a circular curve of radius* $500\,m$*. The distance between the rails is* $1.5\,m$ *and the outside rail is* $0.1\,m$ *above the inside rail. Find the speed that eliminates a sideways force on the wheels for a train on this curve. (Take* $g = 9.8\,ms^{-2}$ *.)*