# 4 Unit Math Homework for Year 12

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| **Student Name**: _____ | **Grade**: _____ |
| **Date**: _____ | **Score**: _____ |

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# 8 Topic 8 — Harder 3 Unit Topics Part 1

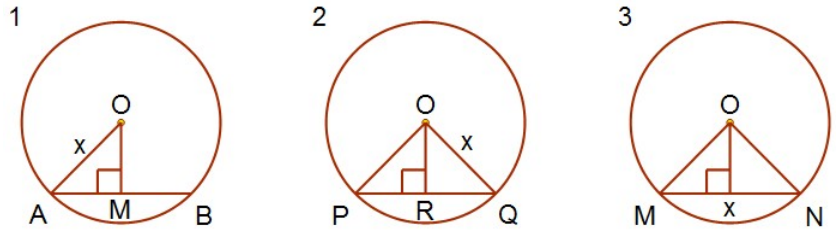
## 8.1 Geometry of Circle

### 8.1.1 Chord Properties

**Chord Properties of Circle 1**:

- A perpendicular drawn to a chord from the centre of a circle bisects the chord, and the perpendicular bisector of a chord passes through the centre.
- The line from the centre of a circle to the midpoint of the chord meets the chord at right angles.
- When two circles intersect, the line joining their centres bisects their common chord at right angles.

**Exercise 8.1.1 Use trigonometry to find x in each diagram (correct to 1 decimals place.).**

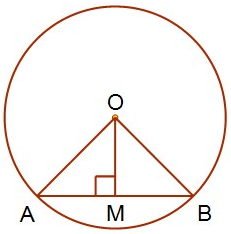



1. *Given that* $\angle AOM = 50°,\ AB = 12\,cm$.

2. *Given that* $\angle POQ = 150°,\ PQ = 15\,cm$.

3. *Given that* $\angle MON = 130°,\ OM = 8.5\,cm$.

**Exercise 8.1.2 For diagram shown below $AO = 26\,cm,\ OM = 10\,cm$. Find the length of $AB$, giving reasons.**




**Exercise 8.1.3**

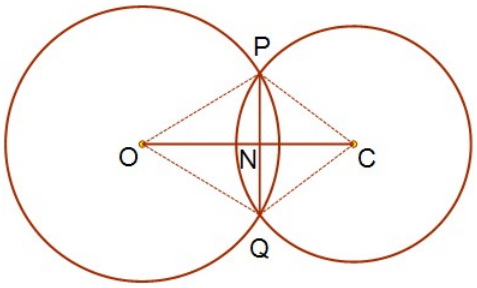
1. *Find the radius of a circle in which a chord of length 14 cm subtends and angle of $70^\circ$ at the centre (correct to 1 decimals place.).*

2. *A chord subtends an angle of $110^\circ$ at the centre of a circle of radius 5.6 cm. Find the length of the chord correct to one decimal place.*

3. *A chord of length 12 cm is drawn on a circle of radius 16 cm. How far this chord from the centre of the circle?*

4. *A chord of length 22 cm has a perpendicular distance of 8 cm from the centre of the circle. What is the radius of the circle?*

**Exercise 8.1.4 These two circles have as their centres points O and C. PQ is the common chord joining the points of intersection of the two circles. N is the point where PQ intersects the line $OC$ which joins the centres.**




1. *Prove that the triangles $POC$ and $QOC$ are congruent.*

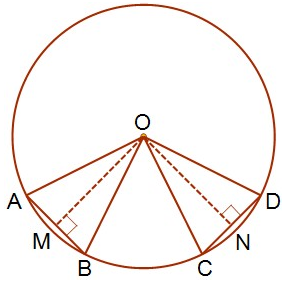
2. *Hence, show that $\angle POC = \angle QOC$.*

3. *Now, prove that the triangles $PON$ and $QON$ are congruent.*

4. *Hence, show that $N$ bisects $PQ$ and that $PQ \perp OC$.*

**Chord Properties of Circles 2**:

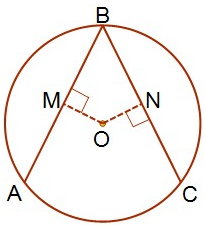
Equal chords of a circle are the same distance from the centre and subtend equal angles at the centre.



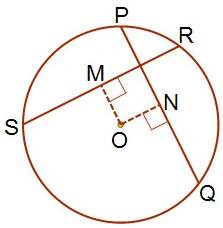

$\angle AOB = \angle COD$ and $OM = ON$.

**Exercise 8.1.5**

1. $AB = BC = 25cm,\ OM = 5.8\,cm$. *Find he length of ON giving reasons.*

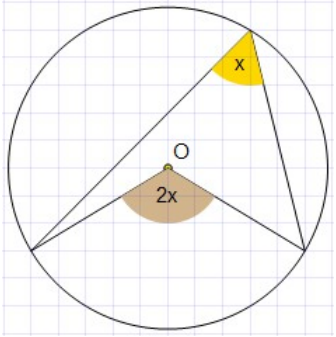



2. $OM = ON = 3.5\,cm, PQ = 12.6\,cm$. *Find the length of $RS$, giving reasons.*

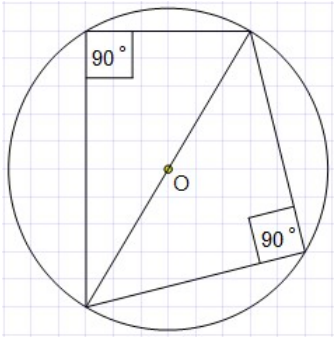


### 8.1.2 Angles in a Circle

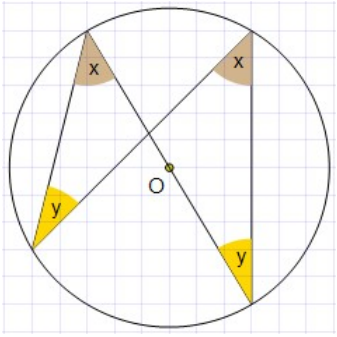
**Definition:** The angle at the centre of a a circle is twice the angle at the circumference standing on the same arc.




**Definition:** The angle in a semicircle is a right angle.



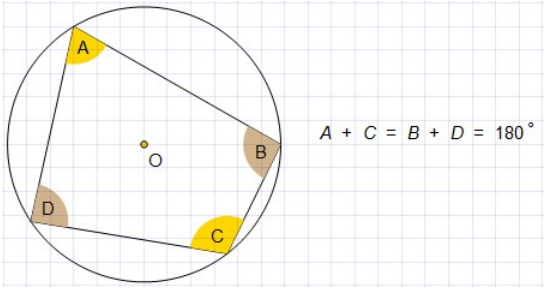

**Definition:** Angles at the circumference of a circle standing on the same arc are equal. In another words, angles in the same segment of a circle are equal.



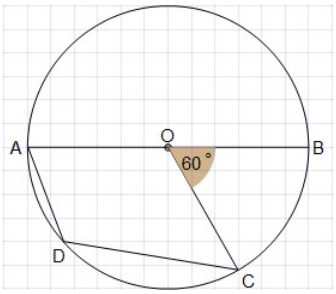
### 8.1.3 Circle Quadrilateral

A cyclic quadrilateral is a quadrilateral whose vertices lie on the circumference of a circle.

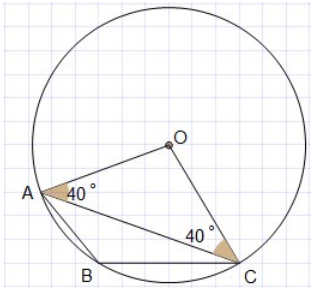
**Definition:** The opposite angles of a cyclic quadrilateral are supplementary (sum to $180°$).




**Exercise 8.1.6 The diagram shows, $AOB$ is the diameter of a circle, centre O. find angle $\angle ADC$.**




**Exercise 8.1.7 If O is the centre of the circle and $\angle CAO = 40°$, find the size of $\angle ABC$**



### 8.1.4 Tangents and Radii

**Definition:** The two tangents form a external point have equal lengths.

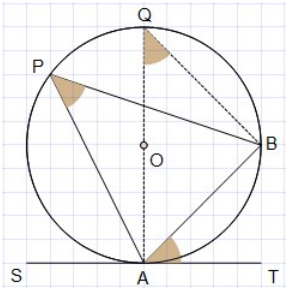
### 8.1.5 Direct and indirect Common Tangents

**Definition:** A common tangent to a pair of circles:

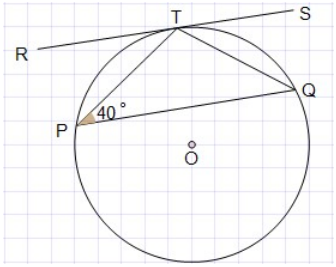
- is called direct, if both circles are on the same side of the tangent,
- is called indirect, if the circles are on the opposite sides of the tangent.

### 8.1.6 The Alternate Segment Theorem

**Definition:** The angle between a tangent to a circle and a chord at the point of contact is equal to any angle in the alternate segment.



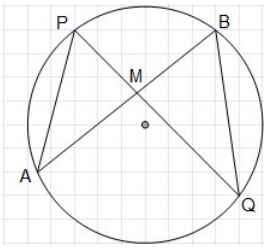

**Exercise 8.1.8 In the figure the tangent TS is parallel to the chord PQ. If $\angle TPQ = 40°$, find the size of $\angle PTQ$.**



### 8.1.7 Intercepts on Intersecting Chords

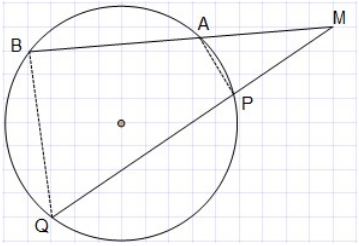
**Definition:** If two chords of a circle intersect, the product of the intercepts on the one chord is equal to the product of the intercepts on the other chord.

$$AM \times MB = PM \times MQ$$

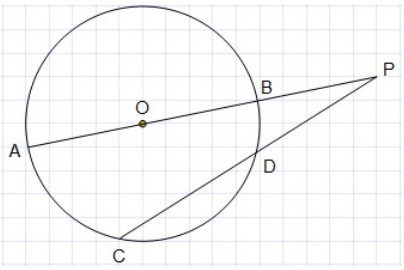



**Definition:** Given a circle and two secants from an external point, the product of the two intervals from the point to the circle on the secant is equal to the product of these two intervals on the other secant.

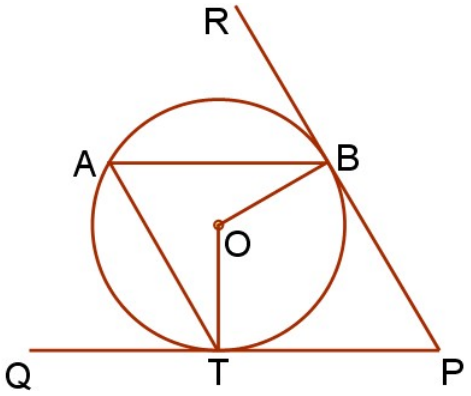
$$AM \times MB = PM \times MQ$$




**Exercise 8.1.9 O is the centre of the circle of radius 9 cm. If $CD = 6\,cm,\ DP = 4\,cm$, find $PB$.**



**Exercise 8.1.10** $O$ **is the centre of the circle.** $BT$ **is a chord that subtends** $\angle BAT$ **at the circumference and** $\angle TOB$ **at the centre.** $PT$ **and** $PB$ **are tangents to the circle.**



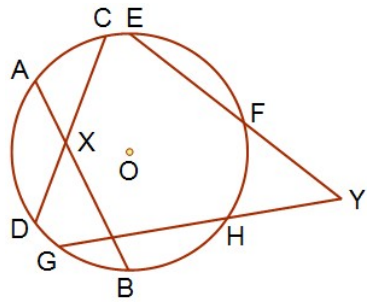

1. *Prove the* $\angle BOT = 2\angle BTP$.

2. *Prove that* $\angle ATQ + \angle RBA + \angle PBT = 180°$.

3. *Prove that* $\angle BPT = 180° - 2\angle BAT$.

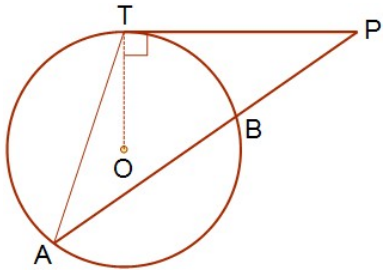
### 8.1.8 Further Circle Properties

- The products of intercepts of intersecting chords or secants are equal.



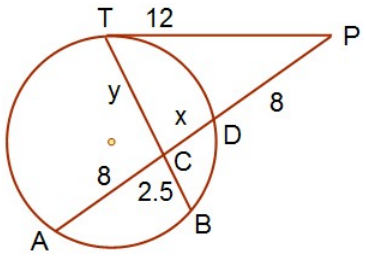

$$AX \times BX = CX \times DX \quad \text{and} \quad EY \times FY = GY \times HY$$

- The square of the length of a tangent is equal to the product of the intercepts of a secant drawn from an external point.

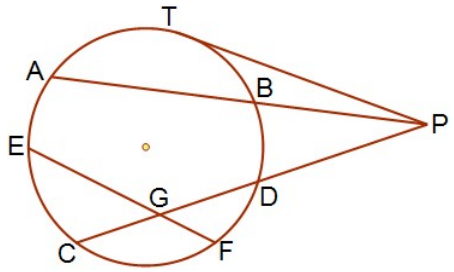



$$(PT)^2 = AP \times PB$$

**Exercise 8.1.11 Find the value of each pronumeral, giving reasons.**



**Exercise 8.1.12 For the diagram given below find:**




1. $PT$ *if* $AB = 9\,cm$ *and* $BP = 3\,cm$

2. $AB$ *if* $BP = 10\,cm$ *and* $PT = 13\,cm$.

3. $CD$ *if* $DP = 5\,cm,\ AB = 8cm$ *and* $BP = 6\,cm$.

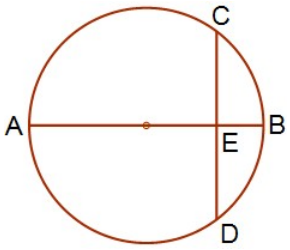
4. $EG$ *if* $GF = 20\,cm,\ CG = 30\,cm$ *and* $DG = 25\,cm$.

5. $CD$ *if* $CG = 16\,cm\ EF = 38\,cm$ *and* $EG = 22\,cm$.

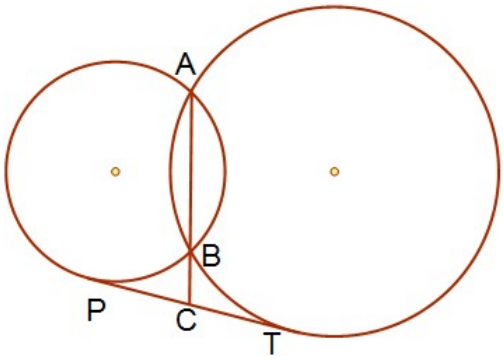
6. $CD$ *if* $TP = 18\,cm$ *and* $DP = 9\,cm$.

**Exercise 8.1.13**

1. *$AB$ is the diameter of a circle. $AB$ bisects a chord $CD$ at the point $E$. Find the length of $CE$ if $AE = 9\,cm$ and $BE = 4\,cm$.*

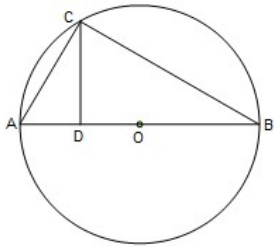



2. *PT is a direct common tangent of the circles drawn. AB is a common chord that has been produced to meet the common tangent at C. Use the 'quare of the tangent' of result to prove that $CP = CT$.*



## 8.2 Pass Exam Questions

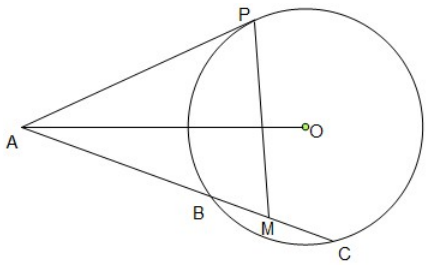
**Exercise 8.2.1** $ADB$ **is a straight line with** $AD = a$ **and** $DB = b$ **. A circle is drawn on** $AB$ **as diameter.** $DC$ **is drawn perpendicular to** $AB$ **to meet this circle at** $C$**.**




1. *Show that* $\triangle ADC|||\triangle CDB$*, and hence show that* $DC = \sqrt{ab}$*.*

2. *Deduce geometrically that if* $a > 0$ *and* $b > 0$*, then* $\sqrt{ab} \leq \frac{a+b}{2}$*.*

**Exercise 8.2.2 Let A is a point outside a circle, AP is the tangent of the circle, P is the point of contact. ABC is a secant. M is a midpoint of BC.**

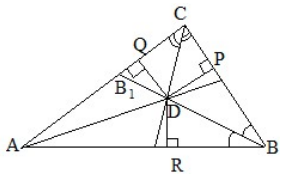



1. *Prove that APOM is a cyclic quadrilateral.*

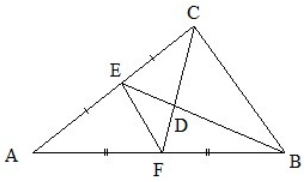
2. *Find the angel sum of $\angle OAM$ and $\angle APM$.*

**Exercise 8.2.3**

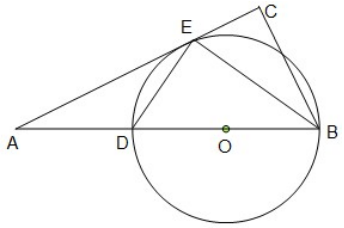
1. *$ABC$ is a triangle. The internal bisectors of $\angle ABC$ and $\angle ACB$ meet at $D$. $DP$, $DQ$ and $DP$ are the perpendiculars from $D$ to $BC$, $CA$ and $AB$ respectively. Show that $DR = DQ$ and deduce that the internal bisectors of the three angles of a triangle are concurrent.*



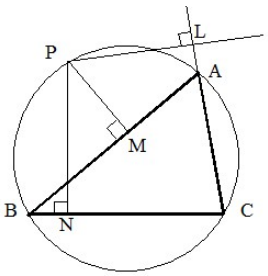

2. *$ABC$ is a triangle. $E$ and $F$ are the midpoints of $AC$ and $AB$ respectively. $BE$ and $CF$ intersects at $D$. Show that the triangle $DEF$ and $DBS$ are similar and hence deduce that the three medians of a triangle are concurrent.*

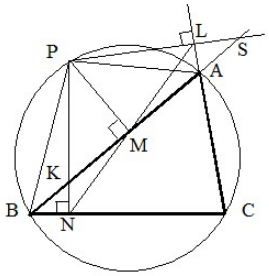


**Exercise 8.2.4 In the diagram, BD is the diameter of the circle, E is a point on the circle , Point A is the intersect of CE produced and and BD produced.** $BC \perp AC$ **. and** $\angle CBE = \angle DBE$**. Prove that AC is a tangent of the circle.**

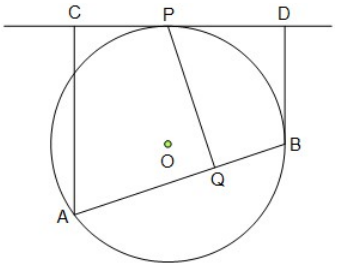



**Exercise 8.2.5** $ABC$ **is a triangle inscribed in a circle.** $P$ **is a point on the minor arc** $AB$**. The points** $L, M,$ **and** $N$ **are the feet of the perpendiculars from** $P$ **to** $CA$ **produced,** $AB$**, and** $BC$ **respectively. Show that** $L$**,** $M$ **and** $N$ **are collinear. (The line** $NL$ **is called the Simpson line.)**

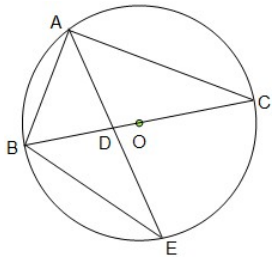


**Exercise 8.2.6 The diagram shows that $AB$ is a chord of the circle and CD is a tangent and meeting the circle at $P$. $AC \perp CD$, $BD \perp CD$ and $PQ \perp AB$. Prove that $PQ^2 = AC \times BD$.**




**Exercise 8.2.7 $ABC$ is a triangle inscribed in a circle. AD bisects the $\angle BAC$ and produced to E, and E is lies on the circle.**

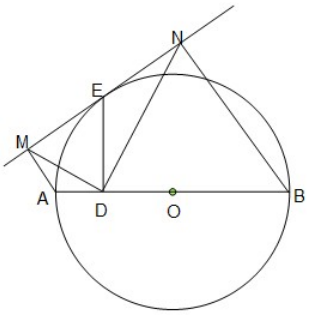



1. *Prove that $\triangle ABE|||\triangle ADC$.*

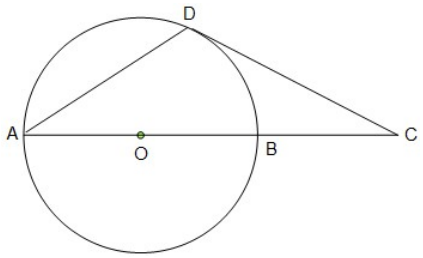
2. *If the area of the triangle ABC is given by $A = \frac{1}{2}AD \times AE$, find the size of the $\angle BAC$.*

**Exercise 8.2.8**

1. *Given that $E$ lies on the circle and $AB$ is the diameter of the circle. $MN$ is a tangent passes through E. $AM \perp MN$, $NB \perp MNandED \perp AB$. prove that $MD \perp ND$.*

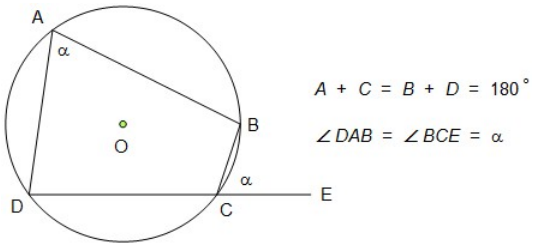



2. *AB is the diameter of a circle, A tangent passes through D produced and meets the AB produced at C. If $DA = DC$, prove that $AB = 2BC$.*

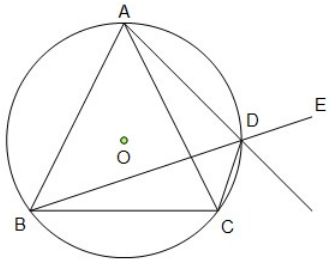


**Cyclic Quadrilaterals:**

- Opposite angles of a cyclic quadrilateral are supplementary.
- An exterior angle of a cyclic quadrilateral equals the opposite interior angle.




**Exercise 8.2.9** $ABC$ **is a triangle inscribed in a circle.** $AB = AC$**, point** $D$ **lies on the circle, produced** $BD$ **to** $E$**.**




1. *Prove that* $AD$ *produced bisects the* $\angle CDE$*.*

2. *If* $\angle BAC = 30°$*, It* $\triangle ABC$*, if the perpendicular height of base BC is* $2 + \sqrt{3}\ cm$*, find the area of the circle inscribed the* $\triangle ABC$*.*