4 Unit Math Homework for Year 12

Student Name: ____________________	**Grade**: ____________________
Date: ____________________	**Score**: ____________________

Table of contents

This edition was printed on June 16, 2017 with worked solutions.
Camera ready copy was prepared with the LaTeX2e typesetting system.

6 Topic 6 — Graphs Part 1

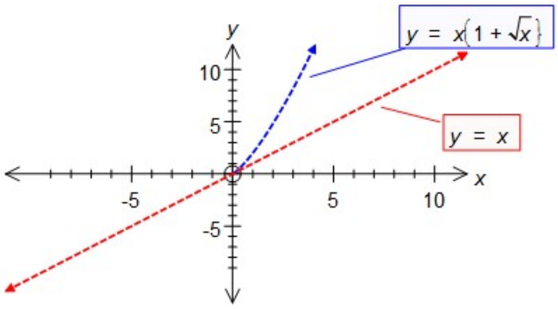
6.1 Critical points on curves

Definition: A critical point on the curve $y = f(x)$ is line at which the derivative $f'(x)$ is not defined.

Example 6.1.1 Sketch (showing critical points) the graphs of

1. $y = x(1+\sqrt{x})$.

Solution:



Domain $\{x : x \geq 0\}$

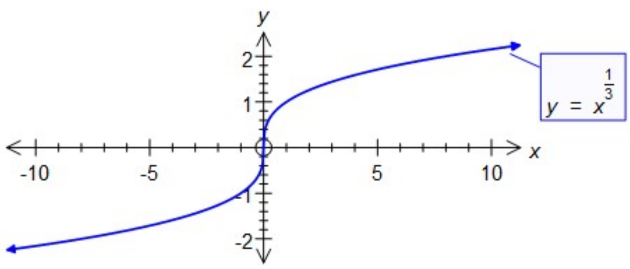
$\dfrac{dy}{dx} = 1 + \dfrac{3}{2}\sqrt{x},\ x > 0.$

$\dfrac{dy}{dx} \ \rightarrow \ 1^{+} as\ x \rightarrow \ 0^{+},$

$\therefore y = x$ *is a tangent line at the critical point* $(0,\ 0)$.

2. $y = x^{\frac{1}{3}}$.

Solution:



$f(x) = x^{\frac{1}{3}} \ \Rightarrow \ f'(x) = \dfrac{dy}{dx} = \dfrac{1}{3}x^{\frac{-2}{3}}$

$\dfrac{dy}{dx}$ *is not defined at x = 0,*

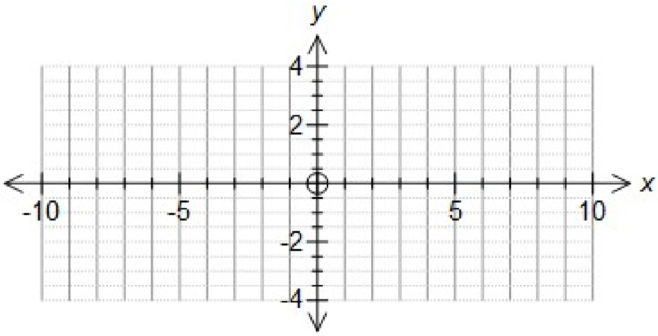
$\therefore (0,\ 0)$ *is a critical point .*

$\dfrac{dy}{dx} \ \rightarrow \ \infty$ *as* $x \ \rightarrow \ 0,$

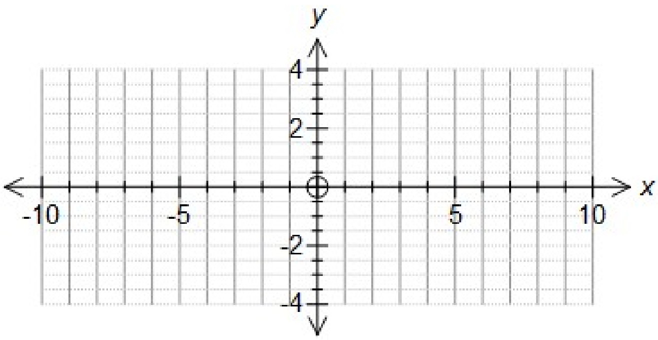
$\therefore$ *the tangent line at* $(0,\ 0)$ *is vertical.*

Exercise 6.1.1 Sketch (showing critical points) the graphs of

1. $y = x^{\frac{1}{2}}$,

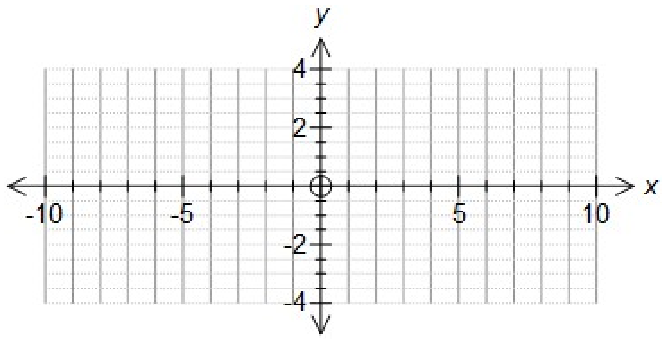


2. $y = x^{\frac{1}{2}} - 3$.

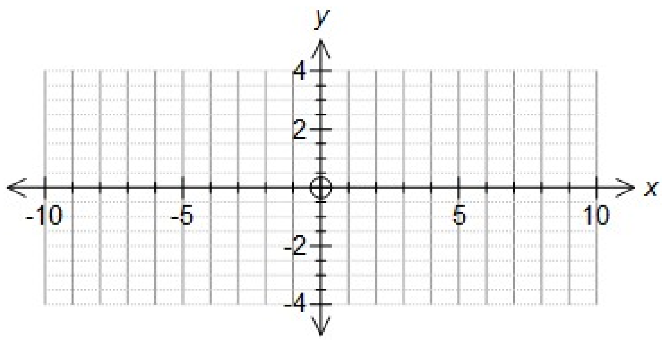


Exercise 6.1.2 Sketch (showing critical points) the graphs of

1. $y = |x + 2|$.

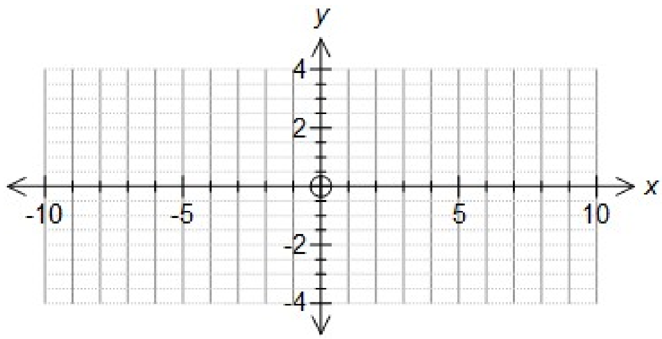


2. $y = |x| + 2$

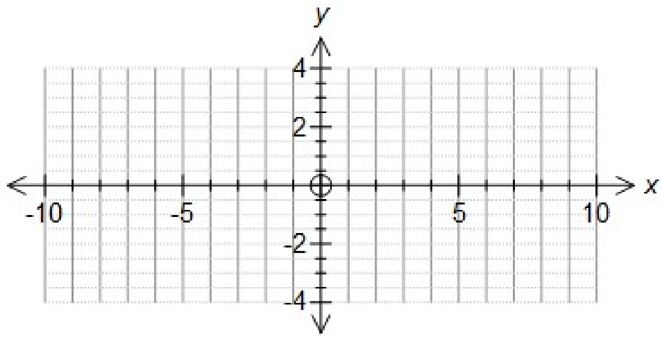


Exercise 6.1.3 Sketch (showing critical points) the graphs of

1. $y = x^2 - |x|$

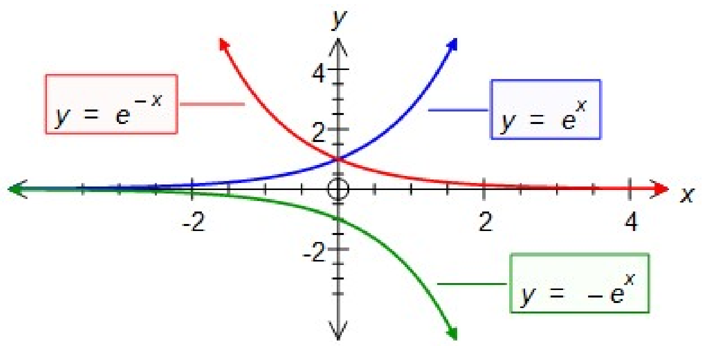


2. $y = |x| + |x - 2|$



6.2 Reflecting graphs in the coordinate axes

Definition: The transformation $x \to -x$ is a reflection in the y -axis.
Hence the graph $y = f(-x)$ is a reflection of $y = f(x)$ in the y-axis.
The transformation $y \to -y$ is a reflection in the x-axis,
Hence the graph $y = -f(x)$ is a reflection of $y = f(x)$ in the x-axis.

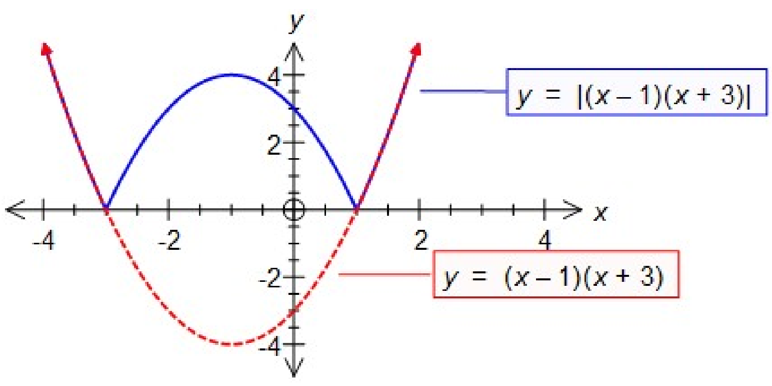


$$|f(x)| = \begin{cases} f(x), & x: \ f(x) \geq 0 \\ -f(x), & x: \ f(x) < 0 \end{cases}$$

Hence those sections of $y = f(x)$ which lie below the x-axis are reflected in the x-axis.

Example 6.2.1 Sketch the graph of $y = |(x-1)(x+3)|$

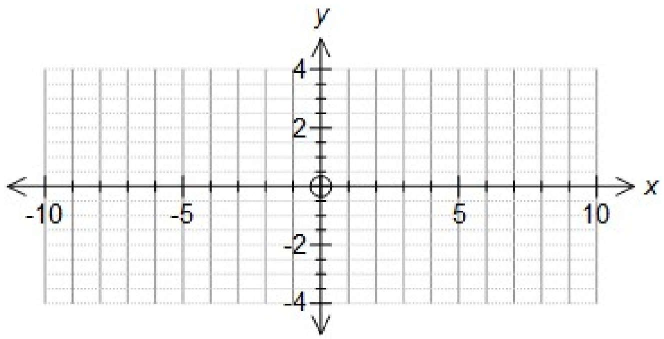
Solution:



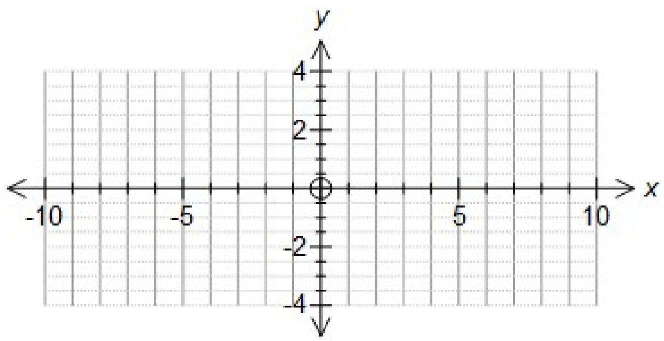
Note that this graph has critical points at $(-3, 0)$ *and* $(1, 0)$*,*
since limiting tangents have different gradients depending on whether the approach these point by tracing the curve from the left or the right.

Exercise 6.2.1

1. *Sketch the graph of* $y = \sin|x|$.

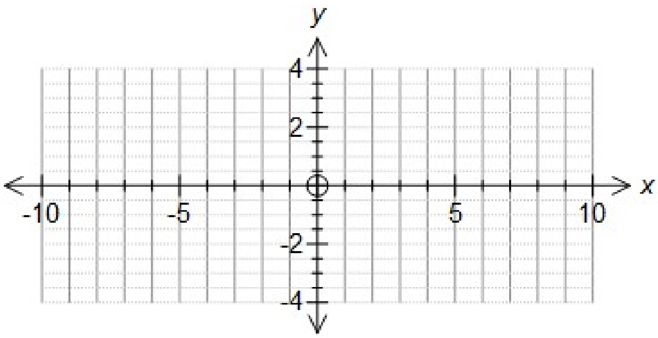


2. *Sketch the graph of* $y = |\sin x|$.

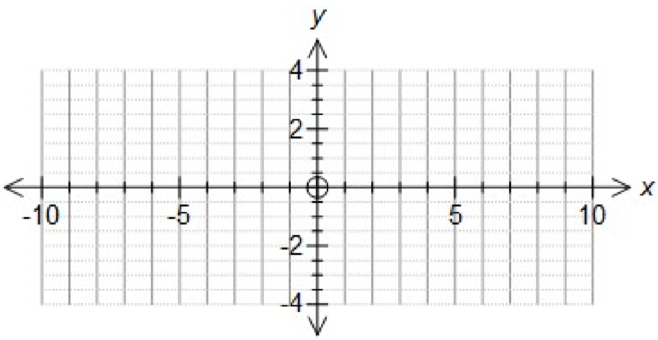


Exercise 6.2.2

1. *For the function* $f(x) = x^2$ *sketch the graphs of:*
   $(a)\ y = f(x),\ (b)\ y = f(-x),\ (c)\ y = -f(x).$

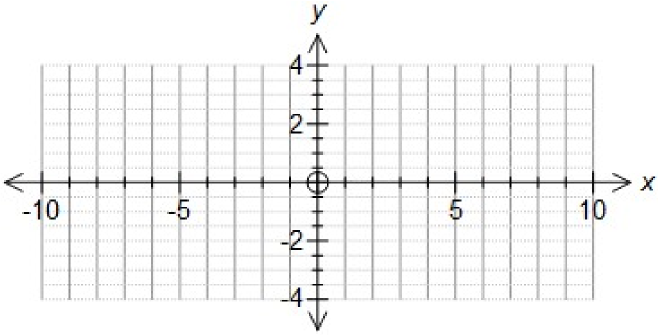


2. *For the function* $f(x) = x^3$ *sketch the graphs of:*
   $(a)\ y = f(x),\ (b)\ y = -f(x),\ (c)\ y = f(-x).$

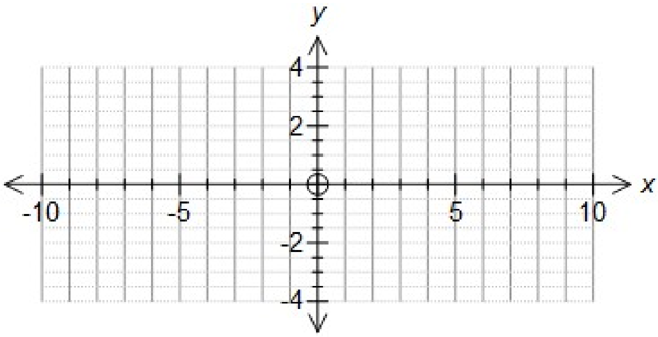


Exercise 6.2.3

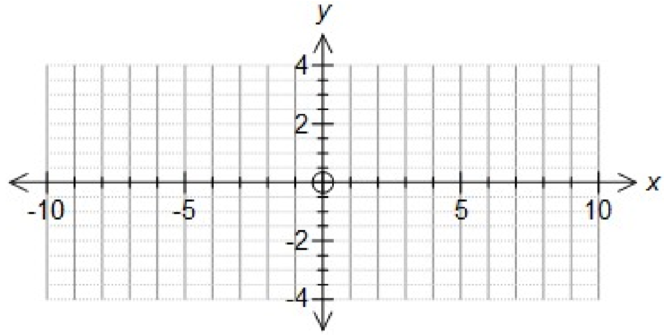
1. *Use the graph of* $y = \ln x$ *to sketch the graphs of:*
   (a) $y = \ln(-x)$, *(b)* $y = -\ln x$.



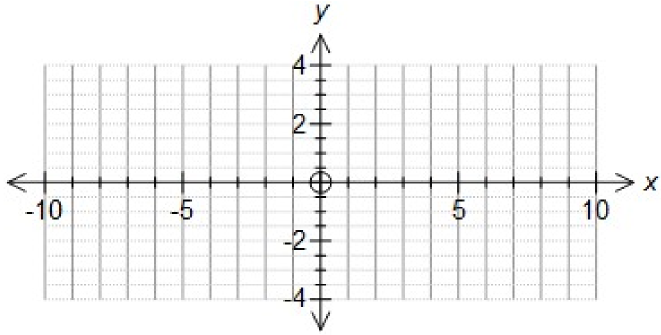
2. *Use the graph of* $y = \ln x$ *to sketch the graphs of:*
   (a) $y = |\ln x|$, *(b)* $y = \ln |x|$.



Exercise 6.2.4 Use the graph of $f(x) = x^3 - 3x$ (an odd function) to sketch the graph of $y = |f(x)|$. Is this the graph of an even function?



Exercise 6.2.5 Use the graph of $f(x) = 4 - x^2$ (an even function) to sketch the graph of $y = |f(x)|$. Is this the graph of an even function?

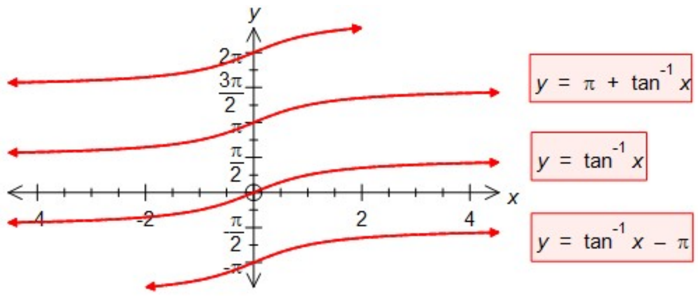


6.3 Translation of graphs: addition and subtraction of ordinates

Definition: The graph $y = f(x) \pm c$ is obtained by translating the graph $y = f(x)$ through c unit up or down .

Example 6.3.1 Sketch $y = \pi + \tan^{-1} x$ and state the domain and range of the function.

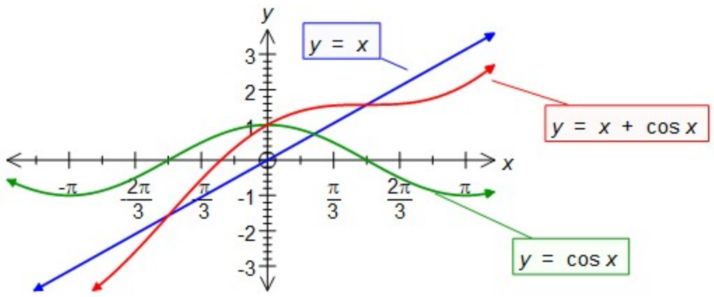
Solution:



Domain all real x Range $\left\{ y : \dfrac{\pi}{2} < y < \dfrac{3\pi}{2} \right\}$.

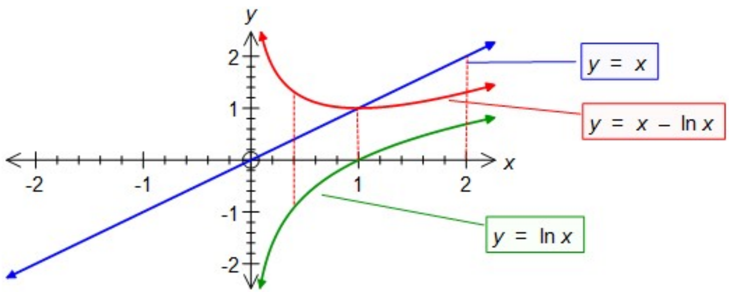
Example 6.3.2 Sketch the graphs $y = x,\ y = \cos x$ on the same axis. Hence sketch $y = x + \cos x$.

Solution:



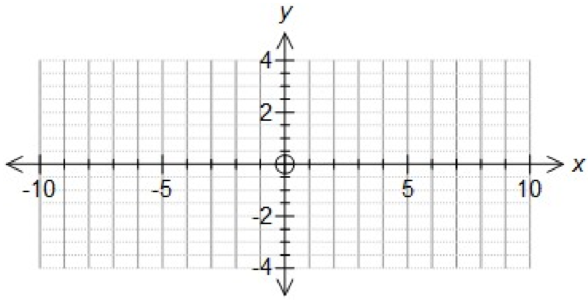
Example 6.3.3 Sketch the graphs $y = x,\ y = \ln x$ on the same axis. Hence sketch $y = x \ln x$.

Solution:

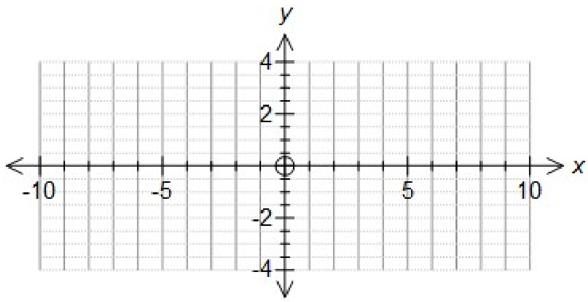


Exercise 6.3.1

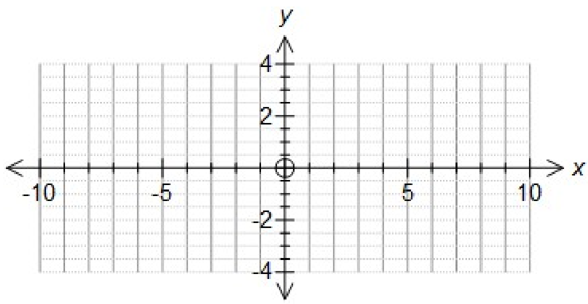
1. *Use the graph of* $y = \cos x$ *to sketch the graphs of:* $(a)\ y = \cos x - 1,\ (b)\ y = \cos(x - \frac{\pi}{2})$.



2. *Use the graph of* $y = x$ *and* $y = e^{-x}$ *to sketch the graph of:* $(a)\ y = x + e^{-x},\ (b)\ y = x - e^{-x}$.

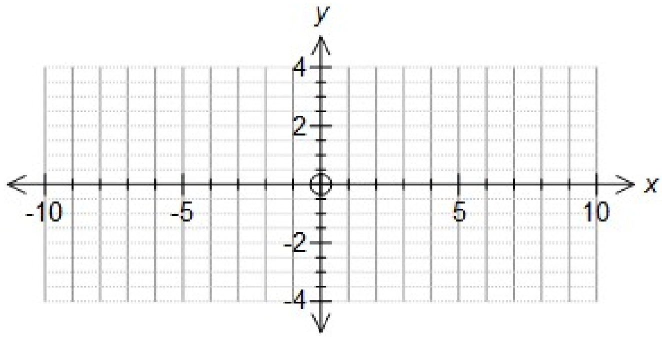


3. *Use the graphs of* $y = \ln x$ *and* $y = \frac{1}{x}$ *to sketch the grapes of:* $(a)\, y = \ln x + \frac{1}{x},\ (b)\, y = \ln x - \frac{1}{x}$.

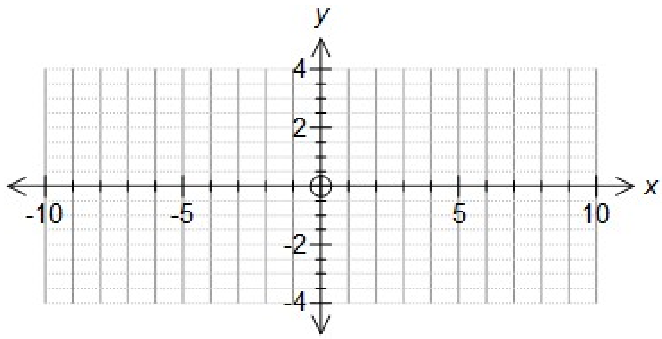


Exercise 6.3.2

1. *Use the graphs* $y = x$ *and* $y = \frac{1}{x}$ *to sketch the graphs of:* $(a)\, y = x + \frac{1}{x}$, $(b)\, y = x - \frac{1}{x}$.



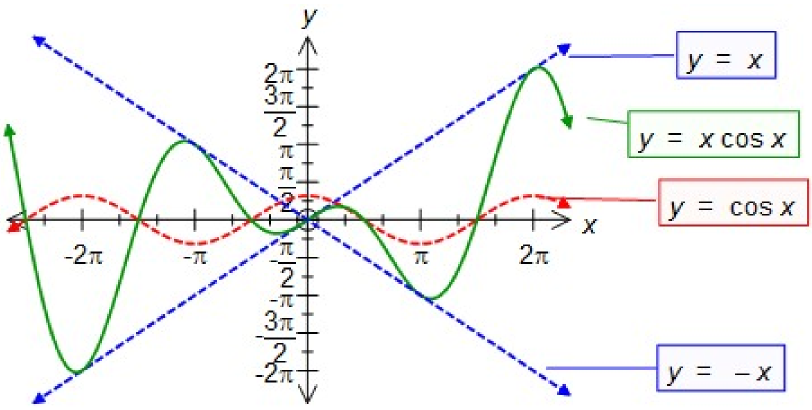
2. *Use the graphs* $y = x$ *and* $y = \frac{1}{x^2}$ *to sketch the graphs of:* $(a)\, y = x + \frac{1}{x^2}$, $(b)\, y = x - \frac{1}{x^2}$.



6.4 Multiplication of ordinates

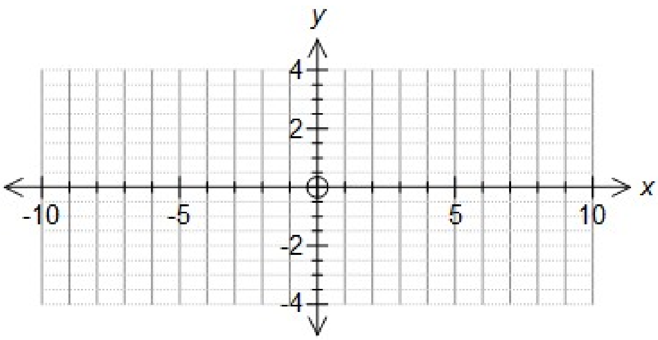
Example 6.4.1 Sketch the graphs $y = x\, y = \cos x$ **on the same axis. Hence sketch** $y = x\cos x$**.**

Solution:



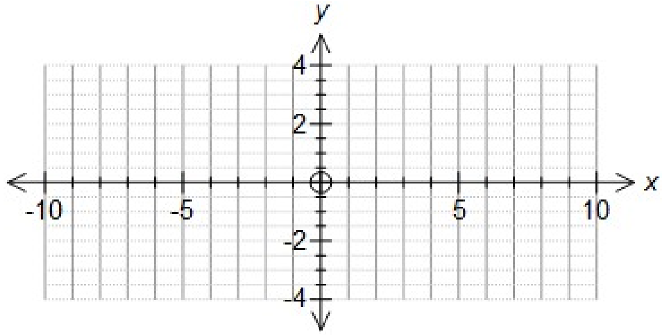
Note tant $y = \cos x$ *is an odd function and hence its graph has point symmetry about the origin.*
For $x \geq 0, -x \leq x\cos x \leq x$ *and hence the graph* $y = x\cos x$ *lies between the lines* $y = \pm x$*, touching these lines when* $\cos x = \pm 1$.

Exercise 6.4.1 Use the graph of $y = sinx$ **to sketch the graph of** $(a)\, y - 2\sin x,\ (b)\, y = \sin 2x$**.**



Exercise 6.4.2

1. *Use the graph* $y = x$ *and* $y = e^{-x}$ *to sketch the graph of* $y = xe^{-x}$.



2. *Use the graph* $y = \ln x$ *and* $y = \frac{1}{x}$ *to sketch the graph of* $y = \frac{\ln x}{x}$.