Year 11 Math Homework

Student Name: ________	**Grade**: ________
Date: ________	**Score**: ________

Table of Contents

This edition was printed on February 24, 2017 with worked solutions.
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9 Year 11 Topic 9 — The Derivative Part 3

9.1 The Notation $\frac{dy}{dx}$ for the Derivative

9.1.1 Small Changes in x and y

- Let $P(x, y)$ be any point on the graph of a function.
- Suppose that x changes by a small amount δx to $x + \delta x$ (Δx to $x + \Delta x$)
- Let y change by a corresponding amount δy to $y + \delta y$ (Δy to $y + \Delta y$).
- Let the new point be $Q(x + \delta x, y + \delta y)$, then the gradient $PQ = \frac{\delta y}{\delta x}$ (rise over run).

When δx is small, the secant PQ is almost the same as the tangent at P, and as before, the derivative is the limit of $\frac{\delta y}{\delta x}$ as $\delta x \to 0$. This is the basis for Leibniz's notation.

Definition: Let δy be the small change in y resulting from a small change δx in x. Then the derivative $\frac{dy}{dx}$ is

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

(Note that the derivative $\frac{dy}{dx}$ is not a fraction, but is the limit of the fraction $\frac{\delta y}{\delta x}$.)

9.1.2 Operator Notation

The derivative $\frac{dy}{dx}$ can also be regarded as the operator $\frac{d}{dx}$ operating on the function y. This operator is also written as D_x.

Example 9.1.1 Differentiate the following functions:

1. $\frac{d}{dx}(x^2 + 2x - 4) = 2x + 2$

   Solution: $\frac{d}{dx}(x^2 + 2x - 4) = 2x + 2$

2. $D_x(2x^2 + 3x - 4)$

   Solution: $D_x(2x^2 + 3x - 4) = 4x + 3$

3. $y = \frac{3x^3+2x^2+x+1}{x}$

   Solution: $y = \frac{3x^3+2x^2+x+1}{x} = 3x^2 + 2x + 1 + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 6x + 2 - \frac{1}{x^2}$

Exercise 9.1.1 Differentiate the following functions:

1. $y = \frac{4x^4+3x^3+2x^2+x+1}{x}$

2. $y = 5x\sqrt{x}$

3. $y = \sqrt[3]{x^2}$

4. $y = \frac{12x-2}{\sqrt{x}}$

5. $y = \frac{6}{x\sqrt{x}}$

9.2 Power of a Linear Function

Power of a Linear Function: $\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$

Example 9.2.1 Use the standard form above to differentiate $\sqrt{9-3x}$.

Solution: $\frac{d}{dx}\sqrt{9-3x} = \frac{-3}{2\sqrt{9-3x}}$ *(with a = -3, b = 9 and n = $\frac{1}{2}$)*

Exercise 9.2.1 Using the standard form above, differentiate the following:

1. $y = (3x-5)^7$

2. $y = \sqrt{25-5x}$

3. $y = \frac{1}{4-2x}$

4. $y = (4x-2)^4$

5. $y = \frac{1}{5x-3}$

9.3 The Chain Rule

Suppose that y is a function of u, where u is a function of x.
The Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Example 9.3.1

1. $y = (x^2 + 2)^5$

Solution: $y = (x^2+2)^5$, *Let* $u = (x^2+2) \Rightarrow y = u^5$
$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5(x^2+2)^4 \times 2x$
$= 10x(x^2+2)^4$

2. $y = 3(5x+8)^6$

Solution: $y = 3(5x+8)^6$, *Let* $u = (5x+8) \Rightarrow y = 3u^6$
$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 18(5x+8)^5 \times 5$
$= 90(5x+8)^5$

Exercise 9.3.1 Use the chain rule to differentiate the following functions:

1. $y = (3x+6)^5$

2. $y = 3(6x+4)^4$

3. $y = \sqrt{36 - x^2}$

9.4 The Product Rule

The derivative of the product of two functions:

Product Rule: If $y = uv$ and u and v are functions of x,
then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$ or $y' = u'v + uv'$

Example 9.4.1 Use the product rule to differentiate the function $y = x^3(x-4)$.

Solution: $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = (x-4)\times(3x^2) + x^3 \times 1 = 3x^3 - 12x^2 + x^3 = 4x^3 - 12x^2$

Exercise 9.4.1 Use the product rule to differentiate the following functions:

1. $y = x(x-8)^3$

2. $y = x^3(2x+3)^2$

3. $y = x\sqrt{x+3}$

4. $y = (2x-4)^4(3x+5)^5$

9.5 The Quotient Rule

The derivative of the quotient of two functions:

The Quotient Rule: If $y = \frac{u}{v}$ and u and v are functions of x

then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ or $y' = \frac{u'v - uv'}{v^2}$

Example 9.5.1 Use the quotient rule to differentiate the function $y = \frac{2x-1}{2x+1}$**.**

Solution: $y = \frac{2x-1}{2x+1} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$= \frac{2(2x+1) - 2(2x-1)}{(2x+1)^2}$$

$$= \frac{4}{(2x+1)^2}$$

Exercise 9.5.1 Use the quotient rule to differentiate the following functions:

1. $y = \frac{x+1}{x-1}$

2. $y = \frac{3-2x}{x+8}$

3. $y = \frac{x^2-6}{x^2+2}$

4. $y = \frac{x^2}{4-x}$

9.6 Practical Exam Questions

Exercise 9.6.1 Differentiate the following functions:

1. $y = (x^2 + 3)(x - 5)$

2. $y = \frac{x - 2\sqrt{x} + 1}{\sqrt{x}}$

3. $y = \frac{4}{x^2\sqrt{x}}$

4. $y = \frac{2}{3x - 1}$

5. $y = (x^3 - 2x)^2$

Exercise 9.6.2 Differentiate the following:

1. $y = 5(3x - 2)^5$

2. $y = (x - 2)(x - 1)^2$

3. $y = x(3 - 2x)^3$

4. $y = \frac{x^2}{2-x}$

5. $y = \frac{3x^2-1}{2x^3+1}$