Year 11 Math Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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9 Year 11 Topic 9 — The Derivative Part 2

9.1 The Derivative

9.1.1 Rules for Differentiation

Rule 1: The Derivative of Constant Function	$y = c$, $\frac{dy}{dx} = 0$ or $y' = 0$

Rule 2: The Derivative of a Linear Function	$y = mx$, $\frac{dy}{dx} = m$

Rule 3: The Derivative of Power of x	$y = x^n$, $\frac{dy}{dx} = nx^{n-1}$

Rule 4: Linear Combination of Function	$y = ax^n$, $\frac{dy}{dx} = anx^{n-1}$

9.1.2 Two Special Forms of Derivatives

Form 1:	The derivative of $\frac{1}{x}$ is $-\frac{1}{x^2}$

Form 2:	The derivative of $\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$

Example 9.1.1 Differentiate the following functions:

1. $f(x) = \frac{5}{16x}$

   Solution: $f'(x) = -\frac{5}{16x^2}$.

2. $f(x) = \sqrt{16x}$

   Solution: $f(x) = \sqrt{16x} = 4\,x^{\frac{1}{2}}$, *so* $f'(x) = 4 \times \frac{1}{2\sqrt{x}} = \frac{2}{\sqrt{x}}$.

3. $f(x) = \frac{12}{x^2}$

   Solution: $f(x) = 12\,x^{-2}$, *so* $f'(x) = 12 \times (-2)x^{-3} = \frac{-24}{x^3}$.

9.1.3 Linear Combinations of Functions

Derivative of a Sum: If $f(x) = g(x) + h(x)$, then $f'(x) = g'(x) + h'(x)$.

Derivative of a Multiple: If $f(x) = kg(x)$, then $f'(x) = kg'(x)$.

Exercise 9.1.1 Differentiate the following functions:

1. $f(x) = 2x^3 - 3x + 5$

2. $f(x) = \frac{4}{x^2} + \frac{4}{x^3}$

3. $f(x) = \frac{1}{3}x^6 - \frac{1}{4}x^2$

4. $f(x) = \frac{4}{x^4} + \frac{4}{x^3}$

5. $f(x) = \sqrt{5x} + \sqrt{7x} + \frac{1}{2x}$

Exercise 9.1.2 Differentiate these functions by first expanding the products:

1. $f(x) = (3x - 2)(2x + 3)$

2. $f(x) = (2x + 1)(2x - 1)$

3. $f(x) = (x^2 + 5)(x - 2)$

4. $f(x) = 2x(x - 4)^2$

5. $f(x) = (2x^2 + 2)^2$

6. $f(x) = (ax - 6)^2$

Exercise 9.1.3 Write these functions using a negative power of x, then differentiate. Give your final answers in fractional form without negative indices.

1. $f(x) = \frac{1}{2x^5}$

2. $f(x) = \frac{a}{bx}$

3. $f(x) = -\frac{3}{5x^5}$

Exercise 9.1.4 Use the fact that the derivative of $\frac{1}{x}$ is $-\frac{1}{x^2}$ to differentiate the following:

1. $f(x) = \frac{3}{x}$

2. $f(x) = \frac{1}{3x}$

3. $f(x) = -\frac{a}{bx}$

Exercise 9.1.5 Use the fact that the derivative of $\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$ to differentiate the following:

1. $f(x) = 5\sqrt{x}$

2. $f(x) = \sqrt{36x}$

3. $f(x) = 3\sqrt{36x}$

4. $f(x) = 2\sqrt{49x}$

5. $f(x) = 12\sqrt{x}$

6. $f(x) = 2\sqrt{x}$

9.1.4 Tangents and Normals to a Curve

- The tangent at a point P is the gradient of the line through the P.
- The normal at P is defined to be the line through P perpendicular to the tangent at P.

Example 9.1.2 Find the equations of the tangent and normal to the curve $f(x) = x^3 - 3x$ at the point $P(2, 2)$ on the curve. Also find the point on the the curve where the tangent is horizonal.

Solution: *$f'(x) = 3x^2 - 3$, so at P(2,2), $f'(2) = 9$,*
So the tangent has gradient 9 and the normal has gradient $-\frac{1}{9}$.
Therefore the tangent is $y - 2 = 9(x - 2)$
$$y = 9x - 16$$
and the normal is $y - 2 = -\frac{1}{9}(x - 2)$
$$y = -\frac{1}{9}x + 2\frac{2}{9}$$
Also, the tangent has gradient zero when $3x^2 - 3 = 0$; so $x = 1$ or $x = -1$
$\therefore$ the tangent is horizontal at $(1,\ 2)$ and $(-1,\ 2)$.

Exercise 9.1.6 Find the points on the graph of $f(x) = x + \frac{1}{x}$ where:

1. *the tangent is horizontal,*

2. *the normal has gradient -2,*

3. *the tangent has angle of inclination 45^o.*

Exercise 9.1.7 Find the equations of the tangent and normal at the point on $y = f(x)$ where x = 3:

1. $f(x) = x^2 + 4x + 4$

2. $f(x) = x^3 - 3x^2 - 6x$

3. $f(x) = 3\sqrt{x}$

Exercise 9.1.8 Find the derivative $\frac{dy}{dx}$ of each of the following functions:

1. $y = 3x^2\sqrt{x} - 2x\sqrt{x}$

2. $y = \frac{3x^2-2x+4}{\sqrt{x}}$

3. $y = (x + \frac{1}{x})^2$

4. $y = \sqrt{x^3}$

5. $y = 4(\sqrt{x} - \frac{1}{\sqrt{x}})^2$

6. $y = 2x^{-2} + \frac{1}{2}x^{-8}$

9.2 Miscellaneous Exercises

Exercise 9.2.1

1. *The first three terms of an arithmetic series are* $-1, +4, +9, +...$

   (a) *Find the 60th term.*

   (b) *Hence, or otherwise, find the sum of the first 60 terms of the series.*

2. *Differentiate the following:*

   (a) $y = -\frac{3}{5x}$

   (b) $y = 2x^3 + 7x$

   (c) $x^{\frac{1}{3}}$

   (d) $y = x^2(2 - 3x + 4x^2)$

Exercise 9.2.2

1. *Solve the equation* $x + \frac{1}{x} = 3$ *(Leave answer in surd form.)*

2. *Express* $\frac{6+\sqrt{2}}{3+\sqrt{2}}$

3. *Solve the following inequations:*

   (a) $\frac{1}{x} > \frac{1}{7}$

   (b) $|x + 1| < 5$

   (c) $|2x - 3| \leq 7$