Year 11 Math Homework

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

Table of contents

This edition was printed on February 19, 2017 with worked solutions.
Camera ready copy was prepared with the LaTeX2e typesetting system.

8 Year 11 Topic 8 — Trigonometry Part 5

8.1 The Sine Rule and the Area Formula

- In any triangle $\triangle ABC$, the ratio of each side to the sine of the opposite angle is constant.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- The area of a triangle is half the product of any two sides and the sine of the included angle.

$$\text{Area of } \triangle ABC = \frac{1}{2} b\, c \sin A = \frac{1}{2}\, a\, c \sin B = \frac{1}{2}\, a\, b \sin C$$

Exercise 8.1.1

1. *In* $\triangle ABC$, $\sin A = \frac{1}{4}$, $\sin B = \frac{2}{3}$, *and* $a = 12$ *cm. Find the value of b.*

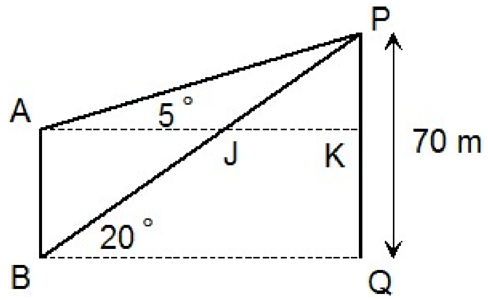
2. *Sketch* $\triangle ABC$ *in which* $a = 2.8$ *cm,* $b = 2.7$ *cm, and* $A = 52°21'$.

   (a) *Find the size of* $\angle B$*, correct to the nearest minute.*

   (b) *Hence find* $\angle C$*,correct to the nearest minute.*

   (c) *Hence find the area of* $\triangle ABC$*, correct to two decimal places.*

Exercise 8.1.2 Two towers AB and PQ stand on level ground. The angles of elevation of the top of the taller tower from the top and bottom of the shorter tower are 5° and 20° respectively. The height of the taller tower is $70\,m$.



1. *Find the size of* $\angle APB$.

2. *Show that* $AB = \frac{BP\,sin\,15^\circ}{\sin\,95^\circ}$.

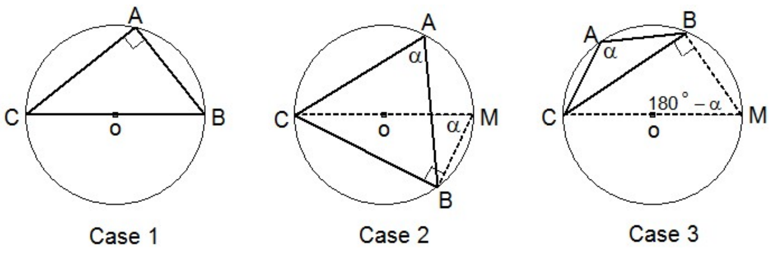
3. *Show that* $BP = \frac{70}{\sin\,20^\circ}$.

4. *Find the height of the shorter tower, correct to the nearest metre.*

8.1.1 The Sine Rule and the Circumcircle

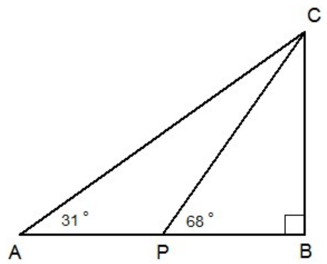
In any triangle, the ratio of each side to the sine of the opposite angle is constant, and this constant is equal to the diameter of the circumcircle of the triangle:

$$\frac{a}{\sin A} = \frac{b}{sin\, B} = \frac{c}{\sin C} = \text{diameter of the circumcircle}$$



- In case 1, a = d and also $\sin\, 90° = 1$.
- In case 2, let $\angle M = \alpha$, $\angle CBM = 90°$, $\sin\, \alpha = \frac{a}{d}$ or $d = \frac{a}{sin\, \alpha}$.
- In case 3, let $\angle M = 180° - \alpha$, since $\sin(180° - \alpha) = \sin\, \alpha$. So $\sin\, \alpha = \frac{a}{d}$ or $d = \frac{a}{\sin\, \alpha}$.

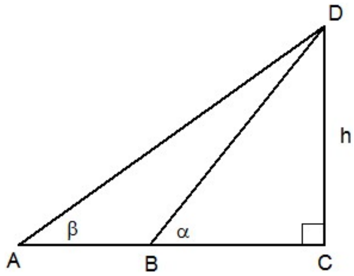
Exercise 8.1.3 In $\triangle ABC$, $\angle B = 90°$ and $\angle A = 31°$. P is a point on AB such that $AP = 20\, cm$ and $\angle CPB = 68°$.



1. *Show that $PC = \frac{20\, \sin 31°}{\sin\, 37°}$.*

2. *Hence find PB, correct to the nearest centimetre.*

Exercise 8.1.4 For the figure given below, $AB = x$ **and** $AC = y$**:**



1. *Show that* $h = \frac{x \sin \alpha \sin \beta}{\sin(\alpha - \beta)}$.

2. *Use the fact that* $\tan \alpha = \frac{h}{y-x}$ *and* $\tan \beta = \frac{h}{y}$ *to show that* $h = \frac{x \tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$.

3. *Combine the expressions in questions (1) and (2) to show* $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$.

4. *Hence find the exact value of* $\sin 15°$.

8.2 The Cosine Rule

- The square of any side of a triangle equals the sum of the squares of the other sides minus twice the product of those sides and the cosine of their included angle:

$$a^2 = b^2 + c^2 - 2\,b\,c\,\cos\,A$$

- The cosine rule with $cos A$ as the subject: $\cos\,A = \frac{b^2+c^2-a^2}{2bc}$

Exercise 8.2.1

1. *In* $\triangle ABC$, $a = 31$ *cm,* $b = 24$ *cm and* $\cos\,C = \frac{59}{62}$. *show that:*

   (a) $c = 11$ *cm.*

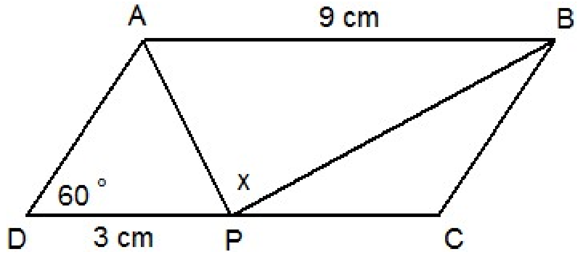
   (b) $A = 120°$

2. *In* $\triangle PQR$, $p = 5\sqrt{3}$ *cm,* $q = 11$ *cm and* $R = 150°$.

   (a) *Find the value of r, correct to nearest centimetre.*

   (b) *Find* $\angle P$, *correct to nearest degree.*

3. *ABCD is a parallelogram in which AB = 9 cm, AD = 3 cm and* $\angle ADC = 60°$. *The point P is the point on CD such that DP = 3 cm.*



(a) *Explain why* $\triangle ADP$ *is equilateral and hence find AP.*

(b) *Use the cosine rule in* $\triangle BCP$ *to find the exact length of BP.*

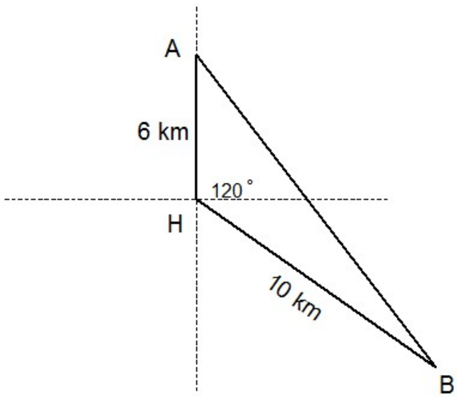
(c) *Let* $\angle APB = x$. *Show that* $\cos x = -\frac{1}{14}\sqrt{7}$.

8.3 Problems Involving General Triangles

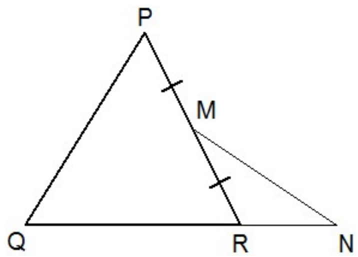
8.3.1 Problems Requiring Two Steps

Exercise 8.3.1

1. *A boat sails 6 km due north from the harbour H to point A and a second boat sails 10 km from H to point B on a bearing of* $120°$. *What is the bearing of B from A, correct to the nearest minute?*

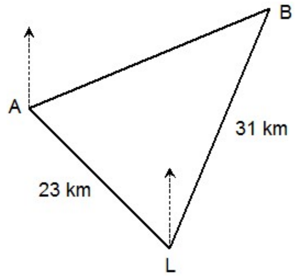


2. *PQR is a equilateral triangle with side lengths 6 cm. M is the midpoint of PR and N is the point in QR produced such that RN = 4. Find MN and hence find* $\angle QNM$, *correct to the nearest minute.*



Exercise 8.3.2

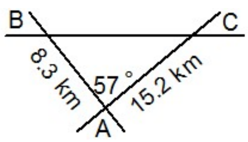
1. *Town A is 23 km from landmark L in the direction $N56°W$, and town B is 31 km from L in the direction $N46°E$.*



   (a) *Find how far town B is from town A (answer to the nearest km).*

   (b) *Find the bearing of town B from town A (answer to the nearest degree).*

2. *AB, BC and CA are straight roads, where AB and AC intersect at $57°$. AB = 8.3 km and AC = 15.2 km. Two cars P_1 and P_2 leave A at the same time. P_1 travels along AB and BC at 80 km/h while P_2 travels along AC at 50 km/h. Which car reaches C first and by how many minutes?*

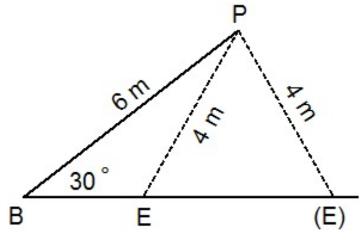


8.3.2 Finding the Third Side in an Ambiguous SAS Situation

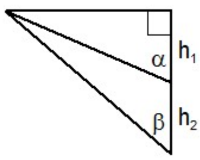
$a^2 = b^2 + c^2 - 2bc \cos A$ can also be written as a quadratic: $c^2 - 2bc \cos A + (b^2 - a^2) = 0$.
This allows the third side to be found in one step in the case of an ambiguous SAS situation (where two sides and a non-included angle are given). For there to be two solutions, the quadratic must have two positive solutions.

Exercise 8.3.3

1. *A tree trunk grows at an angle of* $30°$ *to the ground, and a 4 metre rod is suspended from a point P that is 6 metres along the trunk. Find the maximum and minimum distances of the other end E of the rod from the base B of the tree if E is touching the ground.*



2. *In the diagram shown below, prove that* $h_1 = \frac{h_2 \cos\alpha \sin\beta}{\sin(\alpha-\beta)}$.

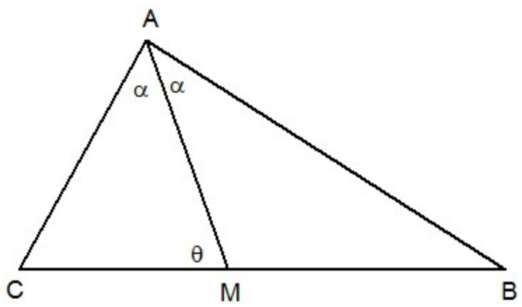


8.3.3 Miscellaneous Exercises

Exercise 8.3.4

1. *The vertical angle of an isosceles triangle is* $35°$ *and its area is 35* cm^2. *Find the length of the equal sides, correct to one decimal place.*

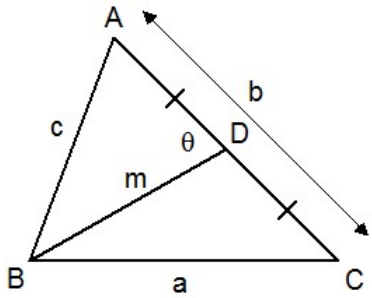
2. *In a triangle ABC, the bisector of angle A meets the opposite side BC at M. Let* $\alpha = \angle CAM = \angle BAM$, *and let* $\theta = \angle CMA$.



   (a) *Explain why* $sin\angle BMA = sin\,\theta$.

   (b) *Hence show that AC : AB = MC : MB.*

Exercise 8.3.5 ABC is a triangle and D is the midpoint of AC. $BD = m$ **and** $\angle ADB = \theta$**.**



1. *Simplify* $\cos(180° - \theta)$.

2. *Show that* $\cos\theta = \frac{4m^2+b^2-4c^2}{4mb}$, *and write down a similar expression for* $\cos(180° - \theta)$.

3. *Hence show that* $a^2 + c^2 = 2m^2 + \frac{1}{2}b^2$.