Year 11 Math Homework

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

Table of contents

This edition was printed on February 19, 2017 with worked solutions.
Camera ready copy was prepared with the LaTeX2e typesetting system.

8 Year 11 Topic 8 — Trigonometry Part 4

8.1 Trigonometry Equations

8.1.1 Quadrants and The Related Angle Method

- Always pay attention to the domain in which the angle can lie.
- If a trigonometric equation involves boundary angles, read the solution off a sketch of the graph.

The Related Angle Method:

1. Draw a quadrant diagram, then draw a ray in each quadrant that the angle could be in.
2. Find the related angle (only work with positive numbers here).
3. Make the angles on the ends of the rays, taking account of any restrictions on x (the domain), and write a conclusion.
4. Try to change any of the three reciprocal functions to the three common function by taking the reciprocal.

Exercise 8.1.1 Solve each of these equations for $0° \leq x \leq 360°$:

1. $\sin\, x = \frac{\sqrt{3}}{2}$

2. $\cos\, x = -\frac{1}{\sqrt{2}}$

3. $\tan\, x = -\sqrt{3}$

Exercise 8.1.2 Solve each of these equations for $0^\circ \le \theta \le 360^\circ$:

1. $cosec\,\theta = -2$

2. $cosec^2\,\theta = 2$

3. $\cot^2\theta = 3.$

4. $\cos^2\theta = \frac{1}{2}$

5. $\sec^2\theta = \frac{4}{3}$

8.1.2 Equations with Compound Angles

- Let u be the compound angle.
- Find the restrictions on u from the given restrictions on x.
- Solve the trigonometric equation for u and then solve for x.

Example 8.1.1 Solve the equation $\tan 2\theta = \sqrt{3}$ **for** $0^\circ \le \theta \le 360^\circ$**.**

Solution: *Let* $u = 2\theta$, $\Rightarrow$ *then* $\tan u = \sqrt{3}$, *where* $0^\circ \le u \le 720^\circ$.
Since $\tan 60^\circ = \sqrt{3}, \Rightarrow$ *the related angle* $u = 60^\circ$, *(*1^{st} *and* 3^{rd} *quadrants).*
So $u = 60^\circ, 240^\circ, 420^\circ$ *or* 600°.
$\therefore\ \theta = 30^\circ,\ 120^\circ,\ 210^\circ$ *or* 300°.

Exercise 8.1.3 Solve each of these equations for $0^\circ \le \alpha \le 360^\circ$**:**

1. $\sin(\alpha - 250^\circ) = \frac{\sqrt{3}}{2}$

2. $\cot(\alpha + 60^\circ) = 1$

3. $cosec\,(\alpha - 75^\circ) = -2$

Exercise 8.1.4 Solve each of these equations for $0^\circ \leq \theta \leq 360^\circ$:

1. $\sin(\theta + 30^\circ) = -\frac{\sqrt{3}}{2}$

2. $\sec 2\theta = 0$

3. $\tan 3\theta = \sqrt{3}$

4. $\cot(\theta - 30^\circ) = 1$

5. $\cos(\theta - 220^\circ) = \frac{\sqrt{3}}{2}$.

8.1.3 Equations Requiring Algebraic Substitutions

- Substitute u to obtain a purely algebraic equation.
- Solve the algebraic equation (it may have more than one solution).
- Solve each of the resulting trigonometric equations.

Example 8.1.2 Solve the equation $\cos^2\theta - \cos\theta = 0$ **for** $0^\circ \leq \theta \leq 360^\circ$**.**

Solution: *Let* $u = \cos\theta, \Rightarrow u^2 - u = 0, \Rightarrow u(u-1) = 0, \Rightarrow u = 0$ *or* $u = 1,$
$\cos\theta = 0, \Rightarrow \theta = 90^\circ;\ \cos\theta = 1, \Rightarrow \theta = 0^\circ$
$\therefore \theta = 0^\circ\ 90^\circ,\ 270^\circ,\ 360^\circ.$

Exercise 8.1.5 Solve for $0^\circ \leq \theta \leq 360^\circ$**, giving solutions correct to the nearest minute where necessary:**

1. $4\sin^3\theta = 3\,sin\,\theta$

2. $4\,cosec^2\,\theta - 4\,cosec\,\theta - 15 = 0$

3. $\tan^2\theta - \tan\theta - 2 = 0$

8.1.4 Equations with More Than One Trigonometric Function

- Use trigonometric identities to produce an equation in only one trigonometric function, then proceed by substitution.
- If all else fails, reduce everything to $sin\,\theta$ and $cos\,\theta$ and hope for the best!

Example 8.1.3 Solve the equation $sec^2\,\theta - 2\,tan\,\theta - 4 = 0$ **for** $0° \le \theta \le 360°$**.**

Solution: *Since* $\tan^2\theta + 1 = \sec^2\theta,\ \ sec^2\,\theta - 2\,\tan\,\theta - 4 = 0,\ \Rightarrow 1 + tan^2\theta - 2\tan\theta - 4 = 0,$
Now $\tan^2\theta - 2\tan\theta - 3 = 0$ *Let* $u = \tan\theta,\ \Rightarrow\ u^2 - 2u - 3 = 0,$
$(u-3)(u+1) = 0,\ \Rightarrow\ u = 3\ or\ u = -1$
$\tan\theta = 3,\ \Rightarrow\ \theta = 71°34'$ *and* $\tan\theta = -1,\ \Rightarrow\ \theta = 45°$ *(related angles)*
$\therefore \theta = 71°34',\ 135°,\ 251°34'\ or\ 315°.$

Exercise 8.1.6 Solve for $0° \le \theta \le 360°$**:**

1. $6\,tan^2\,\theta = 5\,sec\,\theta$

2. $6\,cosec^2\,\theta = cot\,\theta + 8$

Exercise 8.1.7 Solve for $0° \leq \theta \leq 360°$**:**

1. $2\,sin^2\theta + cos\,\theta = 2$

2. $\sec^2\theta + \tan\theta = 1$

3. $\frac{1-\tan^2\theta}{1+\tan^2\theta} + \cos\theta = 0$

4. $(\sqrt{3}+1)\cos^2\theta - 1 = (\sqrt{3}-1)\sin\theta\cos\theta$

8.1.5 Homogeneous Equations

To solve a homogenous equation in $sin\,\theta$ and $cos\,\theta$, divide through by power of $cos\,\theta$ to produce an equation in $tan\,\theta$.

Exercise 8.1.8 Solve each of the following homogeneous equations for $0^\circ \leq \theta \leq 360^\circ$ by dividing both sides by a suitable power of $cos\,\theta$. Give solutions to the nearest minute where necessary:

1. $sin\,\theta = \cos\,\theta$

2. $4\,\sin\,\theta = 3\,cosec\,\theta$

3. $\sqrt{3}\,\sin\,\theta + \cos\,\theta = 0$

4. $\sec\,\theta = 2\,\cos\,\theta$

Exercise 8.1.9 Solve each of the following homogeneous equations for $0^\circ \leq \theta \leq 360^\circ$ by dividing both sides by a suitable power of $cos\,\theta$. Give solutions to the nearest minute where necessary: (Extension)

1. $sin\,\theta = 3\,\cos\,\theta$

2. $5\,\sin^2\,\theta + 8\,\sin\,\theta\,\cos\,\theta = 4\,\cos^2\,\theta$

3. $sin^2\,\theta - 2\,\sin\,\theta\,\cos\,\theta - 8\,\cos^2\,\theta = 0$

4. $sin^3\,\theta + 2\,sin^2\,\theta\,cos\,\theta + sin\,\theta\,\cos^2\,\theta = 0$