Year 11 Math Homework

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

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8 Year 11 Topic 8 — Trigonometry Part 3

8.1 Trigonometric Identities and Elimination

8.1.1 The Three Reciprocal Identities

$$\boxed{cosec\,\theta = \frac{1}{\sin\,\theta}} \quad \boxed{\sec\,\theta = \frac{1}{\cos\,\theta}} \quad \boxed{\cot\,\theta = \frac{1}{\tan\,\theta}}$$

Note:

- Both $\tan\,90^\circ$ and $\tan\,270^\circ$ are undefined, so we cannot use $\cot\,90^\circ = \frac{1}{\tan\,90^\circ}$ or $\cot\,270^\circ = \frac{1}{\tan\,270^\circ}$.
- However, $\cot\,90^\circ = \cot\,270^\circ = 0$.

8.1.2 The Two Ratio Identities

$$\boxed{\tan\,\theta = \frac{\sin\,\theta}{\cos\,\theta}} \quad \boxed{\cot\,\theta = \frac{\cos\,\theta}{\sin\,\theta}}$$

8.1.3 The Three Pythagorean Identities

$$\boxed{\sin^2\,\theta + \cos^2\,\theta = 1} \quad \boxed{\tan^2\,\theta + 1 = \sec^2\,\theta} \quad \boxed{\cot^2\,\theta + 1 = cosec^2\,\theta}$$

8.1.4 The Three Identities for Complementary Angles

$$\boxed{\cos(90^\circ - \theta) = \sin\,\theta} \quad \boxed{\cot(90^\circ - \theta) = \tan\,\theta} \quad \boxed{cosec(90^\circ - \theta) = \sec\,\theta}$$

Exercise 8.1.1 Simplify the following:

1. $1 - \sin^2\,\theta$ ____________________

2. $1 + \cot^2\,\theta$ ____________________

3. $cosec^2\,\theta - 1$ ____________________

4. $\sec^2\,\beta - \tan^2\,\beta$ ____________________

5. $\cot^2\,\beta - cosec^2\,\beta$ ____________________

6. $\frac{1}{\cot\,(90^\circ - \beta)}$ ____________________

Exercise 8.1.2 Prove that:

1. $\tan\theta\cos\theta = \sin\theta$

2. $\cot\theta\,\sin\theta = \cos\theta$

3. $\sin\theta\,\sec\theta = \tan\theta$

4. $\cos\beta\,cosec\,\beta = \cot\beta$

5. $cosec\,\beta\,\cos\beta\,\tan\beta = 1$

6. $\sin\beta\,\cot\beta\,\sec\beta = 1$

Exercise 8.1.3 Prove the identities:

1. $\cos^2\theta\,(\tan^2\theta+1)=1$

2. $2\sin\theta\,\cos\theta+1=(\sin\theta+cos\,\theta)^2$

3. $2\,\tan^2\theta-1=2\sec^2\theta-3$

4. $\cos^2\theta-\sin^2\theta=1-2\,\sin^2\theta$

5. $\cos^4\theta+\cos^2\theta\,\sin^2\theta=\cos^2\theta$

6. $\cos^2\theta\,\tan^2\theta+\sin^2\theta\,\cot^2\theta=1$

Exercise 8.1.4 Prove the following identities:

1. $(\cos\ \beta + \cot\ \beta)\ \sec\ \beta = 1 + cosec\,\beta$

2. $\frac{\sin\ \beta}{cos\,\beta} + \frac{cos\,\beta}{sin\,\beta} = \sec\ \beta\,cosec\,\beta$

3. $\frac{1+\tan^2\ \beta}{1+\cot^2\ \beta} = \tan^2\ \beta$

4. $\frac{1+\cot\ \beta}{1+\tan\ \beta} = \cot\ \beta$

5. $\sin^4\beta - \cos^4\ \beta = \sin^2\ \beta - \cos^2\ \beta$

6. $\frac{1}{1+\sin\ \beta} + \frac{1}{1-\sin\ \beta} = 2\sec^2\ \beta$

Exercise 8.1.5 Prove the following identities: (Extension)

1. $\frac{1}{\sec\alpha - \tan\alpha} - \frac{1}{\sec\alpha + \tan\alpha} = 2\tan\alpha$

2. $\frac{\cos\alpha}{1+\sin\alpha} = \sec\alpha\,(1 - \sin\alpha)$

3. $\frac{2\cos^3\alpha - \cos\alpha}{\sin\alpha\cos^2\alpha - \sin^3\alpha} = \cot\alpha$

4. $\frac{\cos\alpha - \tan\alpha\sin\alpha}{\cos\alpha + \tan\alpha\sin\alpha} = 1 - 2\sin^2\alpha$

5. $\sec\alpha + \tan\alpha + \cot\alpha = \frac{1+\sin\alpha}{\sin\alpha\cos\alpha}$

Exercise 8.1.6

1. *If* $x = a\cos\theta$ *and* $y = a\sin\theta$ *show that* $x^2 + y^2 = a^2$.

2. *If* $x = a\sec\beta$ *and* $y = b\,tan\,\beta$ *show that* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

3. *If* $x = a\cos\alpha - b\sin\alpha$ *and* $y = a\sin\alpha + b\cos\alpha$, *show that* $x^2 + y^2 = a^2 + b^2$.

Exercise 8.1.7 Eliminate θ from each pair of equations:

1. $x = a\cos\theta$ *and* $y = b\sin\theta$.

2. $x = 2 + \cos\theta$ *and* $y = 1 + \sin\theta$.

3. $x = \sin\theta + \cos\theta$ *and* $y = \sin\theta - \cos\theta$.

Exercise 8.1.8 Prove the identities:

1. $(\sin\ \alpha + \cos\ \alpha)(\sec\ \alpha + cosec\, \alpha) = 2 + \tan\ \alpha + \cot\ \alpha$

2. $\frac{1}{1+\tan^2\ \alpha} - \frac{1}{1+\sec^2\ \alpha} = \frac{\cos^4\ \alpha}{1+cos^2\, \alpha}$

3. $\frac{\cos\ \alpha}{1-\tan\ \alpha} + \frac{\sin\ \alpha}{1-\cot\ \alpha} = \sin\ \alpha + \cos\ \alpha$

4. $\frac{1}{\sec\ \alpha+\tan\ \alpha} = \sec\ \alpha - \tan\ \alpha = \frac{\cos\ \alpha}{1+\sin\ \alpha}$

Exercise 8.1.9 Prove the following identities: (Extension)

1. $\frac{1}{\cot\theta - \cos\theta} = \frac{\tan\theta}{1-\sin\theta} = \frac{\sin\theta + \sin^2\theta}{\cos^3\theta}$

2. $\sin^2\theta\,(1 + n\cot^2\theta) + \cos^2\theta\,(1 + n\tan^2\theta) = n + 1$

3. $\frac{(\sin^2\theta - \cos^2\theta)(1 - \sin\theta\cos\theta)}{\cos\theta\,(\sec\theta - cosec\,\theta)(\sin^3\theta + \cos^3\theta)} = \sin\theta$

4. $\frac{1 + cosec^2 A\tan^2 C}{1 + cosec^2 B\tan^2 C} = \frac{1 + \cot^2 A\sin^2 C}{1 + \cot^2 B\sin^2 C}$

8.2 Practical Exam Questions

Exercise 8.2.1 Prove the following identities:

1. $\frac{1-2\sin 2\alpha\cos 2\alpha}{\cos^2 2\alpha-\sin^2 2\alpha} = \frac{1-\tan 2\alpha}{1+\tan 2\alpha}$.

2. *Given that* $\tan^2\alpha = 2\tan^2\beta + 1$, *prove that* $\sin^2\beta = 2\sin^2\alpha - 1$.

3. *If* $\sin\theta = \frac{\sqrt{5}-1}{4}$, *evaluate* $\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta} + \frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$.

Exercise 8.2.2

1. *If* $\tan\theta = 2$, *evaluate* $\sin^2\theta + \sin\theta\cos\theta - 2\cos^2\theta$.

2. *If* $\sin\theta$ *and* $\cos\theta$ *are two roots of the function* $2x^2 - (\sqrt{3}+1)x + m = 0$, *where* $0 \leq \theta \leq 2\pi$.

   (a) *Evaluate* $\frac{\sin\theta}{1+\frac{1}{\tan\theta}} + \frac{\cos\theta}{1+\tan\theta}$.

   (b) *Find the value of m.*

   (c) *and hence find the roots of the function.*

1. If $\tan\theta = \sqrt{3}$, evaluate $\frac{1}{\sin\theta\cos\theta}$.

2. Prove that $\frac{\cos\theta}{1+\sin\theta} - \frac{\sin\theta}{1+\cos\theta} = \frac{2(\cos\theta - \sin\theta)}{1+\sin\theta+\cos\theta}$

3. If $\sin\theta + 3\cos\theta = 2$, evaluate $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$.