Year 11 Math Homework

Student Name: ________________	**Grade**: ________________
Date: ________________	**Score**: ________________

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8 Year 11 Topic 8 — Trigonometry Part 2

8.1 The Quadrant, the Related Angle and the Sign

8.1.1 The Quadrant and the Related Angle

- The quadrant of θ is the quadrant (1, 2, 3 and 4) in which the ray lies.
- The related angle of θ is the acute angle between the ray and the x-axis.

8.1.2 The Signs of the Trigonometric Functions

- Signs of the trigonometric function follow the rule: All Stations To Central".
  - Where A (All), S (Sin), T(Tan) or C(Cosine) are the only function/s positive.

Exercise 8.1.1

1. *Use the ASTC rule to determine the sign (+ or -) of each of these trigonometric ratios:*
   (a) $sin\,50^\circ$ __________
   (b) $cot\,145^\circ$ __________
   (c) $\sec(-40^\circ)$ __________
   (d) $\tan\ 420^\circ$ __________
   (e) $cosec\,200^\circ$ __________
   (f) $\cos\ 340^\circ$ __________
2. *Find the related angle for each of the following:*
   (a) 320° __________
   (b) -80° __________
   (c) -420° __________
   (d) 580° __________
3. *Write each trigonometric ratio as the ratio of an acute angle with the correct sign attached:*
   (a) $sec\,1020^\circ$ __________
   (b) $sec\,190^\circ$ __________
   (c) $cosec\,320^\circ$ __________
   (d) $tan\,500^\circ$ __________

Exercise 8.1.2

1. *Find the exact value of the following:*

   (a) $\sin\ 240^\circ\ \cos 150^\circ - \sin\ 150^\circ\ \cos\ 240^\circ$.

   (b) $3\ \tan\ 210^\circ \sec\ 210^\circ - \sin\ 330^\circ\ cot\, 135^\circ - \cos\ 150^\circ\ cosec\, 240^\circ$.

2. *Prove the following:*

   (a) $\sin\ 420^\circ\ \cos\ 405^\circ + \cos\ 420^\circ\ \sin\ 405^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$.

   (b) $\frac{\sin\ 135^\circ - \cos\ 120^\circ}{sin\, 135^\circ + \cos\ 120^\circ} = 3 + 2\sqrt{2}$.

Exercise 8.1.3 Show that the following relationships satisfy the given values:

1. $\cos\ 3\theta = 4\ \cos^3\ \theta - 3\ \cos\ \theta$, *where* $\theta = 225°$.

2. $\tan\ 2\theta = \frac{2\tan\ \theta}{1-\tan^2\ \theta}$, *where* $\theta = 135°$.

3. $\sin(A + B) = \sin A \cos B + \cos A \sin B$, *where* $A = 300°$ *and* $B = 240°$.

Exercise 8.1.4 If $\tan^2\theta + 2\sec^2\theta = 5$**, find the value of** $\sin^2\theta$**.**

8.1.3 The Angle and the Related Angle

The trigonometric functions of any angle θ are the same as the trigonometric function of its related angle, apart from a possible change of sign. (Note: The sign is found using the ASTC diagram.)

8.1.4 Evaluating the Trigonometric Function at Any Angle

- Place the ray in the correct quadrant and use the ASTC rule to work out the sign of the answer.
- Find the related angle and work out the value of the trigonometric function at the related angle.

8.1.5 General Angles with Pronumerals

$\sin(180° - A) = \sin A$	$\sin(180° + A) = -\sin A$	$\sin(360° - A) = -\sin A$
$\cos(180° - A) = -\cos A$	$\cos(180° + A) = -\cos A$	$\cos(360° - A) = \cos A$
$\tan(180° - A) = -\tan A$	$\tan(180° + A) = \tan A$	$\tan(360° - A) = -\tan A$

Exercise 8.1.5 Write as a trigonometric ratio of A with the correct sign attached:

1. $\tan(180° + A)$ ____________________

2. $\sec(360° - A)$ ____________________

3. $\sec(180° + A)$ ____________________

4. $\tan(-A)$ ____________________

5. $\sec(-A)$ ____________________

Exercise 8.1.6 Extension questions:

1. Write as a trigonometric ratio of θ *with the correct sign attached:*

(a) $\sin(90° + \theta)$ ____________________

(b) $\sin(270° - \theta)$ ____________________

(c) $\sec(270° - \theta)$ ____________________

2. Simplify:

(a) $\cos(180° - \theta)\sec\theta$ ____________________

(b) $\sin(90° - \theta)\, sec(90° + \theta)$ ____________________

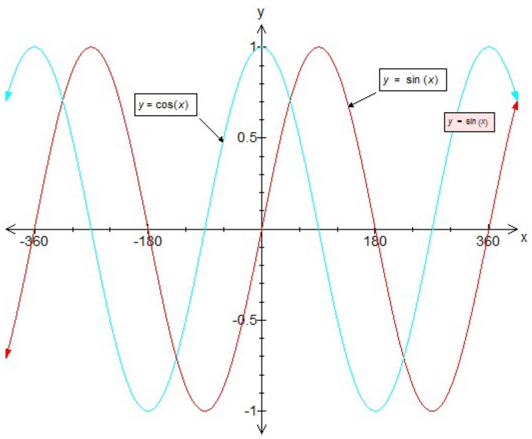
(c) $\cot(180° + \theta)\cos(270° - \theta)$ ____________________

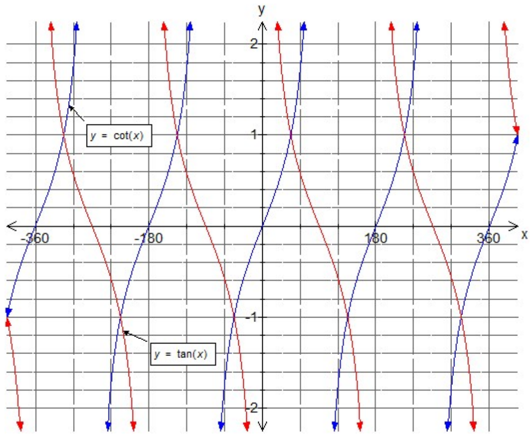
8.1.6 Specifying a Point in Terms of r and θ

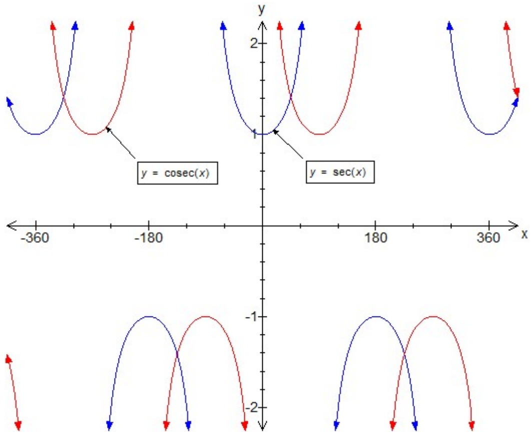
If the definitions of $\sin\,\theta$ and $\cos\,\theta$ are rewritten with x and y as the subject:

$$x = r\ \cos\,\theta, \text{ and } y = r\,sin\,\theta$$

8.1.7 The Graphs of the Six Trigonometric Functions







8.2 Given One Trigonometric Function, Find Another

Draw a circle diagram and use Pythagoras' theorem to find whether x, y or r are missing.

Example 8.2.1 Given that $\sin\theta = \frac{3}{5}$**, find** $\cos\theta$ **and** $\tan\theta$**.**

Solution: $\sin\theta = \frac{3}{5} > 0$ *it must be in the* 1^{st} *or* 2^{nd} *quadrant.*
Since $\sin\theta = \frac{3}{5}$, *another side must be* $\sqrt{5^2 - 3^2} = 4$
$\therefore \cos\theta = \pm\frac{4}{5}$, *and* $\tan\theta = \pm\frac{3}{4}$

Exercise 8.2.1

1. *Given that* $\cos\ \alpha = \frac{3}{5}$, *and* α *is acute, find the possible values of* $\sin\ \alpha$ *and* $\tan\ \alpha$.

2. *Given that* $\tan\ \alpha = -\frac{5}{12}$, *and* α *is obtuse, find* $\sin\ \alpha$ *and* $\sec\ \alpha$.

3. *Given that* $\sin\ \alpha = \frac{8}{17}$, *find the possible values of* $\cos\ \alpha$ *and* $\cot\ \alpha$.

4. *If* $180° < \theta < 270°$ *and* $\sin\theta = p$, *express* $\cos\theta$ *in terms of* p.

Example 8.2.2 Given that $\cot\ x = -\frac{3}{5}$**, Ffind the possible values of** $cosec\, x$ **and** $\sec\ x$**.**

Solution: $\cot x = -\frac{3}{5}, \;\Rightarrow\; \tan x = -\frac{5}{3}$, *and it is in the* 2^{nd} *and* 4^{th} *quadrant.*
other side is $\sqrt{5^2+3^2} = \sqrt{34}$
$\therefore \sin x = \pm\frac{5}{\sqrt{34}}, \;\Rightarrow\; cosecx = \pm\frac{\sqrt{34}}{5}$ *and* $\sec x = \pm\frac{\sqrt{34}}{3}$

Exercise 8.2.2

1. *Given that* $\cot\ \beta = \frac{3}{2}$ *and* $\sin\ \beta < 0$*, find* $\cos\ \beta$*.*

2. *If* $\tan\ \beta = 2$*, find the possible values of* $cosec\,\beta$*.*

3. *Suppose that* $\sin\ \beta = 1$*. find* $sec\,\beta$*.*

4. *If* $\tan\theta = -\frac{4}{3}$ *and* $90^\circ < \theta < 180^\circ$*, find the value of* $\cot\theta, \sin\theta\ and\ \cos\theta$*.*

Example 8.2.3 Suppose that $\cos x = \frac{1}{2}$ **and** x **is acute. Find** $cosec\, x$ **and** $\cot x$**.**

Solution:

$$\sin^2 x + \cos^2 x = 1, \;\Rightarrow\; \sin x = \sqrt{1 - \cos^2 x}$$
$$\therefore \sin x = \sqrt{1 - (\frac{1}{2})^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$
$$\therefore cosec\, x = \frac{1}{\sin x} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$
$$Now\, \tan x = \frac{\sin x}{\cos x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$
$$\therefore \cot x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Exercise 8.2.3

1. *Given that* $\sec\, x = -3$ *and* $180° < x < 360°$, *find* $cosec\, x$.

2. *Suppose that* $\cos x = \frac{2}{3}$. *Find the possible values of* $\sin\, x$ *and* $\cot\, x$.

3. *If* $\sin\theta = \frac{1-t^2}{1+t^2}$ *and* θ *is acute, find expressions for* $\cos\theta$ *and* $\cot\theta$.

Exercise 8.2.4

1. *Given that* $\sin\theta = \frac{x}{y}$, *with* θ *obtuse and* x *and* y *both positive, find the expressions for* $\cos\theta$ *and* $tan\,\theta$.

2. *If* $\tan\ \theta = k$, *where* $k > 0$, *find the possible values of* $\sin\ \theta$ *and* $\sec\ \theta$.

3. *Prove the algebraic identity* $(1 - x^2)^2 + 4x^2 = (1 + x^2)^2$.

4. *If* $\sin\theta = k$ *and* θ *is obtuse, find an expression for* $\tan(90^\circ + \theta)$.

5. *If* $\sec\theta = p + \frac{1}{4p}$, *prove that* $\sec\theta + \tan\theta = 2p$ *or* $\frac{1}{2p}$.