Year 11 Math Homework

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

Table of contents

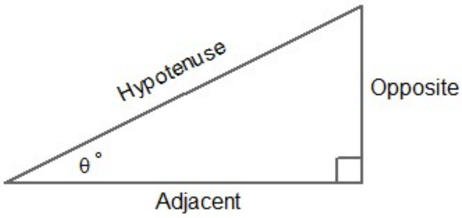
This edition was printed on February 18, 2017 with worked solutions.
Camera ready copy was prepared with the LaTeX2e typesetting system.

8 Year 11 Topic 8 — Trigonometry Part 1

8.1 Trigonometry with Right Triangles

8.1.1 The Definition of the Trigonometric Functions

The definitions of the trigonometric ratio are:



$\sin\,\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{O}{H}$	$\cos\,\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{A}{H}$	$\tan\,\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{O}{A}$
$cosec\,\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$	$\sec\,\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$	$\cot\,\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

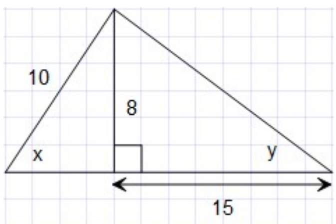
8.1.2 Special Angles

The exact values for the trigonometric ratios are summarised in the table shown below:

θ	$30°$	$45°$	$60°$	θ	$30°$	$45°$	$60°$	θ	$30°$	$45°$	$60°$
$\sin\,\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\cos\,\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\tan\,\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$cosec\,\theta$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	$\sec\,\theta$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\cot\,\theta$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

Exercise 8.1.1

1. *Use Pythagoras' theorem to find the third side of in each of the right triangles in the diagram shown below:*



2. *Write down the value of:*

   (a) $\cos\,y$ ____________________

   (b) $\sin\,x$ ____________________

   (c) $\cot\,y$ ____________________

   (d) $\sec\,x$ ____________________

   (e) $cosec\,y$ ____________________

Exercise 8.1.2 Without using a calculator, find the value of:

1. $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

2. $cosec^2\, 30^\circ - \cot^2 30^\circ$

Exercise 8.1.3 Without using a calculator, show that:

1. $2 \sin 30^\circ \cos 30^\circ = \sin 60^\circ$.

2. $\cos^2 60^\circ - \cos^2 30^\circ = -\frac{1}{2}$

Exercise 8.1.4 Suppose that β is an acute angle and $\sec \beta = \frac{\sqrt{11}}{3}$:

1. *Find the exact value of $cosec\, \beta$.*

2. *Find the exact value of $\cot \beta$.*

3. *Show that $cosec^2\, \beta - \cot^2 \beta = 1$.*

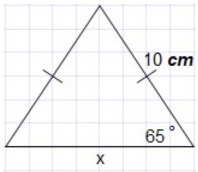
8.1.3 Finding an Unknown Side of a Triangle

To find an unknown side of a right triangle:

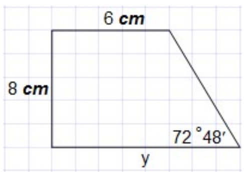
- Start by writing $\frac{\text{unknown side}}{\text{known side}} = \ \ldots$ (place the unknown on the top left).
- Complete the RHS with $\sin\,\theta$, $\cos\,\theta$, $\tan\,\theta$ or their respective reciprocal.

Exercise 8.1.5 Find the value of each pronumeral, correct to three decimal places:

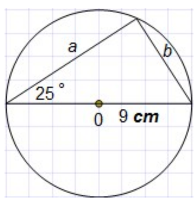
1. *Find the value of **x**.*



2. *Find the value of **y**.*



3. *Find the values of **a** and **b**.*



8.1.4 Finding an Unknown Angle of a Triangle

To find an angle when given two sides of a right triangle, work out one of $\cos\theta$, $sin\,\theta$ or $tan\,\theta$ is known.

Exercise 8.1.6 Answer to four significant figures or to the nearest minute:

1. *A triangle has sides of 7 cm, 7 cm and 5 cm. What are the size of its angles?*

2. *An isosceles triangle has base angles of $76°$. What is the ratio of base to side length?*

3. *The diagonals of a rectangle meet at $35°$,. Find the ratio of the length to the breadth.*

4. *The diagonals of a rhombus are 16 cm and 10 cm. Find the vertex angles to the nearest degree.*

5. *One vertex angle of a rhombus is $25°$. Find the ratio of the diagonals correct to 2 decimal places.*

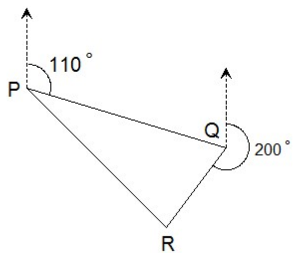
8.1.5 Compass Bearings and True Bearings

- Compass bearings specify direction in terms of four cardinal directions: North, South, East and West.
- Any other direction is given by indicating the deviation from North or South towards the East or West (e.g. $N30°E$, $S45°W$).
- True bearings are measured clockwise from North.

Exercise 8.1.7

1. *From a ship sailing due north, a lighthouse is observed to be on a bearing of* $42°$. *Later when the ship is 2 nautical miles from the lighthouse, the bearing of the lighthouse from the ship is* $148°$. *Find, correct to three significant figures, the distance of the lighthouse from the initial point of the observation.*

2. *A ship leaves port P and travels 150 nautical miles to port Q on a bearing of* $110°$. *It then travels 120 nautical miles to port R on a bearing of* $200°$.



(a) *Explain why* $\angle PQR = 90°$.

(b) *Find, to the nearest degree, the bearing of port R from port P.*

8.1.6 Angles of Elevation and Depression

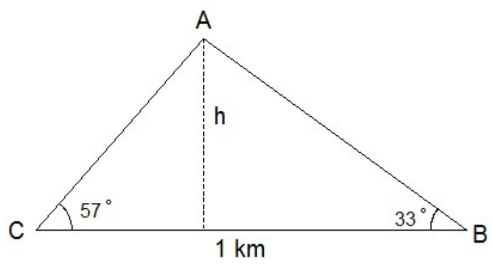
- Angles of elevation and depression are always measured from a horizontal plane.
- They are always acute angles.

Exercise 8.1.8

1. *A ladder of length 5 metres is placed on level ground against a vertical wall. If the foot of the ladder is 1.5 metres from the base of the wall, find to the nearest degree, the angle at which the ladder is inclined to the ground.*

2. *Find, to the nearest degree, the angle of depression of a boat 200 metres out to sea, from the top of a vertical cliff of height 40 metres.*

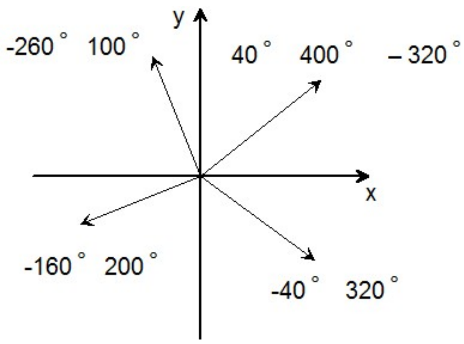
3. *From the ends of a straight horizontal road 1 km long, a balloon directly above the road is observed to have angles of elevation of* $57°$ *and* $33°$ *respectively. Find, correct to the nearest metre, the height of the balloon above the road.*



8.2 Trigonometric Functions of a General Angle

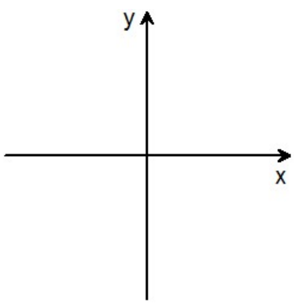
8.2.1 Putting a General Angle on the Cartesian Plane

- To find the ray corresponding to θ, rotate the positive half of the x-axis through an angle θ in the anticlockwise direction.
- To each angle, there corresponds exactly one ray.
- To each ray, there corresponds infinitely many angles, all differing from each other by multiples of $360°$.

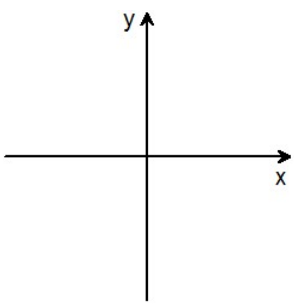


Exercise 8.2.1 On a number plane, draw rays representing the following angles:

1. *(a)* $50°$, *(b)* $120°$, *(c)* $290°$, *(d)* $680°$.



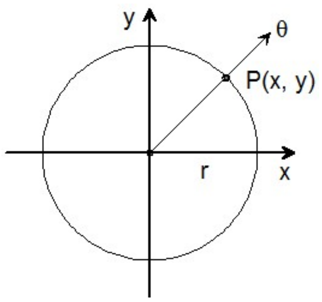
2. *(c)* $-40°$, *(b)* $-150°$, *(c)* $-280°$, *(d)* $-560°$.



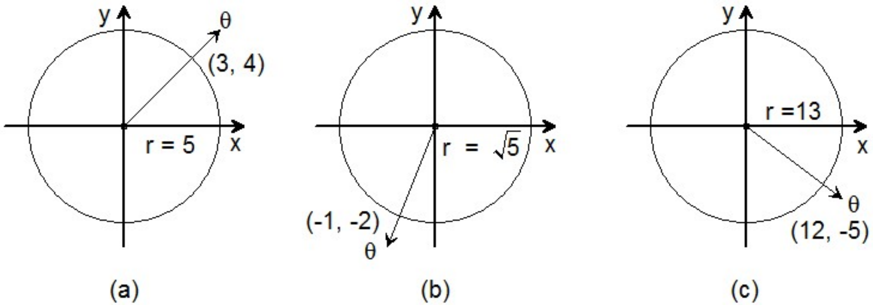
8.2.2 The Definitions of the Trigonometric Functions

Suppose that θ is any angle. Construct the ray corresponding to θ, and construct a circle with the origin as the centre and any positive radius **r**. Let the ray and the circle intersect at the point **P(x, y)**. We can now define the six trigonometric functions:

$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$
$cosec\,\theta = \frac{r}{y}$	$\sec\theta = \frac{r}{x}$	$\cot\theta = \frac{x}{y}$



Exercise 8.2.2 Write down the values of the six trigonometric ratios of the $\angle\theta$ in each diagram:



(a) $\sin\theta =?$, $\cos\theta =?$, $\tan\theta =?$, $cosec\,\theta =?$, $\sec\theta =?$, *and* $\cot\theta =?$,

(b) $\sin\theta =?$, $\cos\theta =?$, $\tan\theta =?$, $cosec\,\theta =?$, $\sec\theta =?$, *and* $\cot\theta =?$,

(c) $\sin\theta =?$, $\cos\theta =?$, $\tan\theta =?$, $cosec\,\theta =?$, $\sec\theta =?$, *and* $\cot\theta =?$,

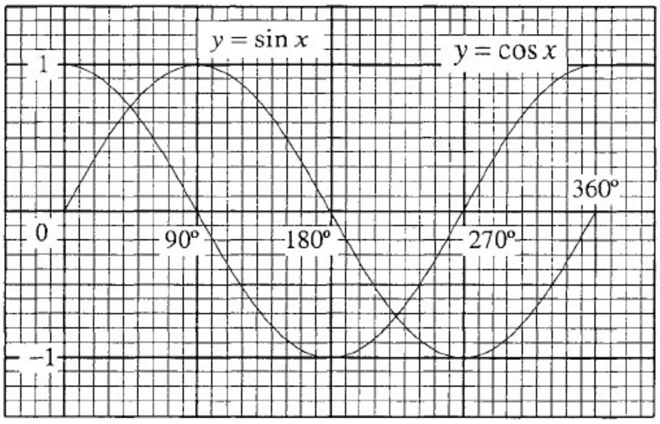
8.2.3 Boundary Angles

- Integer multiples of $90°$, that is ... $-90°, 0°, 90°, 180°$.. are called boundary angles because they lie on the boundaries between quadrants.
- The values of the trigonometric functions at these boundary angles are not always defined.
- When they are defined, they only have the values of -1, 0 and 1.

8.2.4 The Domain of the Trigonometric Functions

- $\sin\ \theta$ and $\cos\ \theta$ are defined for all angles θ.
- $\tan\ \theta$ and $\sec\ \theta$ are undefined for $\theta = ... -90°, 90°, 270°, 540°, ...$
- $\cot\ \theta$ and $cosec\,\theta$ are undefined for $\theta = ... -180°, 0°, 180°, 360°, ...$

Exercise 8.2.3



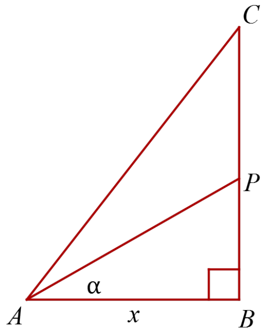
1. *Read off the diagram shown below and find the value of:*
   (i) $\cos\ 60°$, *(ii)* $\sin\ 72°$ *(iii)* $\cos\ 144°$, *(iv)* $\sin 234°$.

2. *Find from the graphs, two values of x between* $0°$ *and* $260°$ *for which:*
   (i) $\sin\ x = 0.5$, *(ii)* $\cos\ x = 0.6$, *(ii)* $\sin\ x = -0.4$, *(iv)* $\cos\ = -0.8$.

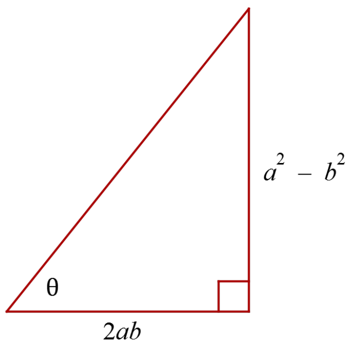
8.3 Theoretical Exercises on Right Triangles (Extension)

Exercise 8.3.1

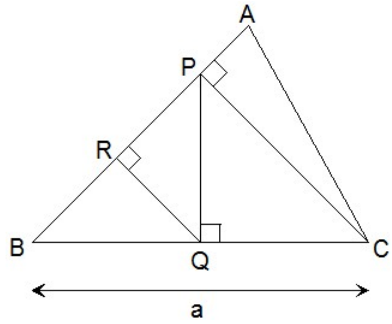
1. *In the diagram shown below, $\triangle ABC$ is a right triangle and P is the midpoint of BC. If $\angle PAB = \alpha$, show that $BC = 2x\ \tan\ \alpha$.*



2. *Prove the algebraic identity $(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$. Hence show that* $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$.



Exercise 8.3.2 In triangle ABC, lines CP, PQ and QR are drawn perpendicular to AB, BC and AB respectively.



1. *Explain why* $\angle RBQ = \angle RQP = \angle QPC$.

2. *Show that* $QR = a\ \sin\ B\ \cos^2\ B$.