Year 11 Math Homework

Student Name: ______	**Grade**: ______
Date: ______	**Score**: ______

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7 Year 11 Topic 7 — Coordinate Geometry Part 3

7.1 Coordinate Geometry – Straight Lines

7.1.1 Distance between Parallel Lines

- *Perpendicular Distance Formula:* $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Exercise 7.1.1

1. *Write down the equation of a line through the origin with gradient* m.

2. *Write down the distance from this line to the point (3, 1).*

3. *If the line is a tangent to the circle* $(x-3)^2+(y-1)^2=4$, *show that* m *satisfies the equation* $5m^2-6m-3=0$.

4. *Find the possible values of* m *and hence the equation of the two tangents.*

Exercise 7.1.2 The point $P(x, y)$ is equidistant from lines $\ell_1 : 2x+y-3 = 0$ and $\ell_2 : x-2y+1 = 0$, which intersect at A.

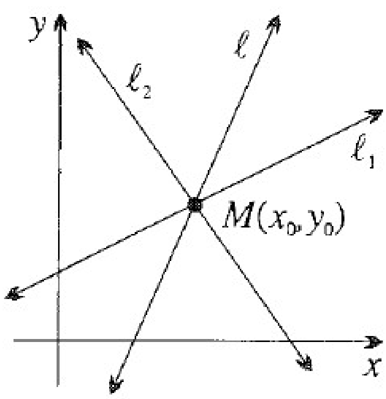
1. *Use the distance formula to show that* $|2x + y - 3| = |x - 2y + 1|$.

2. *Hence find the equations of the lines that bisect the angles at* A.

7.1.2 Lines through the Intersection of Two Given Lines

Suppose that $\ell_1 : a_1x + b_1y + c_1 = 0$ and $\ell_2 : a_2x + b_2y + c_2 = 0$ are two lines intersecting at a point $M(x_0, y_0)$:

- *Line through the Intersection of Two Lines:*
  $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$; where k is a constant.



Exercise 7.1.3 Write down the equation of a line through the intersection M of the lines $\ell_1 : x + 2y - 6 = 0$ and $\ell_2 : 3x - 2y - 6 = 0$. Hence, without finding the point of intersection, find the line through M that:

1. *passes through P(2, -1),*

2. *has gradient 2,*

3. *is horizontal,*

4. *is vertical,*

5. *hence find the coordinates of intersect point M.*

Exercise 7.1.4 The lines $2x + y - 5 = 0$ and $x - y + 2 = 0$ intersect at A.

1. *Write down the general equation of a line through A and show that it can be written in the form* $(2 + k)x + (1 - k)y + (2k - 5) = 0$

2. *Find the value of k that makes the coefficient of x zero and hence find the equation of the horizontal line through A.*

3. *Find the value of k that makes the coefficient of y zero, and hence find the equation of the vertical line through A.*

4. *Write down the coordinate of A.*

Exercise 7.1.5

1. *Find general form of a line through the intersection M of $x - 2y + 5 = 0$ and $x + y + 2 = 0$.*

2. *Show that the gradient of the line is $\frac{1+k}{2-k}$.*

3. *Hence find the equation of the lines through M:*

   (a) *parallel to $3x + 4y = 5$,*

   (b) *perpendicular to $2x - 5y + 4 = 0$,*

   (c) *has gradient 3.*

Exercise 7.1.6 Show that the three lines $\ell_1 : 2x - 3y + 13 = 0$, $\ell_2 : x + y - 1 = 0$ and $\ell_3 : 4x + 3y - 1 = 0$ are concurrent by the following method:

1. *Without finding any point of intersection, find the equation of the line through the intersection of ℓ_1 and ℓ_2 parallel to ℓ_3.*

2. *Show that this line is the same line as ℓ_3.*

Exercise 7.1.7

1. *Use the perpendicular distance formula to show that A(-2, 3) is equidistant from the two lines $\ell_1 : x - 3y + 1 = 0$ and $\ell_2 : 3x + y - 7 = 0$.*

2. *Hence, without finding their point of intersection, find the equation of the line through A that bisects the angle between the two lines.*

7.2 Coordinate Methods in Geometry

- *Median:* A median of a triangle is the interval from a vertex of the triangle to the midpoint of the opposite side.
- *Altitude:* An altitude of a triangle is the perpendicular from a vertex of the triangle to the opposite side

Exercise 7.2.1 Triangle OBA has vertices at the origin O, $A(3,0)$ and $B(0,4)$. C is a point on AB such that OC is perpendicular to AB.

1. *Find the equations of AB and OC and hence find the coordinates of C.*

2. *Find the lengths OA, AB, OC, BC and AC.*

3. *Thus confirm these important corollaries for a right-angled triangle:*

   (a) $OC^2 = AC \times BC$.

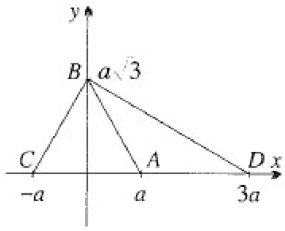
   (b) $OA^2 = AC \times AB$.

Exercise 7.2.2 A triangle has vertices at $A(1, -3)$, $B(3, 3)$ and $C(-3, 1)$.

1. *Find the coordinates of P and Q, the midpoints of AB and BC respectively.*

2. *Show that PQ is parallel to AC and that $PQ = \frac{1}{2}AC$.*

Exercise 7.2.3 The diagram below shows the points A, B, C and D on the number plane.



1. *Show that $\triangle ABC$ is equilateral.*

2. *Show that $\triangle ABD$ is isosceles, with $AB = AD$.*

3. *Show that $AB^2 = \frac{1}{3}BD^2$.*

Exercise 7.2.4 It is true in general that the perpendicular bisectors of the sides of a triangle are concurrent, and that the point of intersection (called the circumcentre) is the centre of a circle through all three vertices (calls the circumcircle). Prove that result in general by placing the vertices at $A(2a, 0)$, $B(-2a, 0)$ and $C(2b, 2c)$, and proceeding as follows:

1. *Find the gradients of AB, BC and CA, and hence find the equations of the three perpendicular bisectors.*

2. *Find the intersection M and any two perpendicular bisectors, and show that it lies on the third.*

3. *Explain why the circumcentre must be equidistant from each vertex.*