Year 11 Math Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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7 Year 11 Topic 7 — Coordinate Geometry Part 1

7.1 Coordinate Geometry — Straight Line

7.1.1 Gradient of a Straight Line

Suppose that $P(x_1, y_1)$ and $Q(x_2, y_x)$ are two distinct points in the number plane:

- *Gradient Formula:* gradient of PQ, $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Two lines are parallel if and only if they have the same gradient, $m_1 = m_2$.
  (or are both vertical)
- Two lines are perpendicular if and only the product of their gradient is -1.
  (ie; $m_1 \times m_2 = -1$, or $m_1 = -\frac{1}{m_2}$)
- *Angle of Inclination:* Gradient $m = \tan\alpha$
- *Testing for Collinear Points:* Distinct points are collinear if they all lie on the same line.

Exercise 7.1.1 Find the gradient of interval AB, then find the gradient of a line perpendicular to it:

1. *A(-5, -2), B(3, 2)*

2. *A(3, 6), B(6, 5)*

3. *A(-a, b), B(3a, -b)*

Exercise 7.1.2 Find the points A and B where each line meets the x-axis and y-axis respectively. Hence find the gradient of AB and its angle of inclination α (to the nearest degree).

1. $y = -\frac{1}{2}x + 1$

2. $\frac{x}{3} - \frac{y}{2} = 1$

3. $4x - 5y - 20 = 0$

4. $2x + 3y = 12$

Exercise 7.1.3 Find the gradients of PQ and QR, and hence determine whether P, Q and R are collinear:

1. *P(-2, 7), Q(1, 1) and R(4, -6)*

2. *P(-5, -4), Q(-2, -2) and R(1, 0)*

Exercise 7.1.4 Given $A = (2, 3)$, write down the coordinates of any three points P such that AP has gradient 2.

Exercise 7.1.5 For the triangle, $A(4, 3)$, $B(3, -2)$, and $C(-1, 2)$. Show that the perpendicular bisector of BC passes through A. What kind of triangle is $\triangle ABC$?

Exercise 7.1.6 On a number plane, mark the origin O and the points A(2, 1) and B(3, -1).

1. *Find the gradients of OA and AB and hence show that they are perpendicular.*

2. *Show that OA = AB.*

3. *Find the midpoint D of OB.*

4. *Given that D is the midpoint of AC, find the coordinates of C.*

5. *What shape best describes quadrilateral OABC?*

Exercise 7.1.7

1. *Show the straight line that passes through the points A(-1, 0) and B(-5, 6) is parallel to the line* $2y + 3x = 5$.

2. *Find the equation of the line perpendicular to the line* $2y + 3x = 5$*, that passes through the midpoint of AB in question (1).*

Exercise 7.1.8 The points O(0, 0), P(4, 0) and Q(x, y) form a right angle at Q and PQ = 1 unit.

1. *Form a pair of equations for x and y.*

2. *Solve them simultaneously to find the coordinates of the two possible locations of Q.*

7.1.2 Equations of a Straight Line

- *Vertical Lines:* The vertical line through $P(a, b)$ has equation $x = a$.
- *Horizontal Lines:* The horizontal line through $P(a, b)$ has equation $y = b$.
- *Gradient-intercept Form:* $y = mx + b$
- *General Form:* $ax + by + c = 0$
- *Point and Gradient Form:* $y - y_1 = m(x - x_1)$

Exercise 7.1.9

1. *Write down the gradient and y-intercept of each line:*

   (a) $y = \frac{1}{4}x - 2$

   (b) $y = 5 - 2x$

2. *Find the gradient of each line below and hence find the gradient-intercept form of a line passing through A(0, 4) and (i) parallel to it, (ii) perpendicular to it:*

   (a) $5x - 2y - 1 = 0$

   (b) $3x + 4y - 5 = 0$

Exercise 7.1.10

1. *Consider the two lines* $\ell_1 : 3x - y + 4 = 0$ *and* $\ell_2 : x + ky + 5 = 0$. *Find the value of k if:*

   (a) ℓ_1 *is parallel to* ℓ_2.

   (b) ℓ_1 *is perpendicular to* ℓ_2.

2. *The line with equation* $y = 3x - 6$ *is rotated in the x-y plane through a right angle about its point of intersection with the x-axis. Find the equation of the line after the transformation.*

3. *Show that the line with equation* $2x - y = 5$ *is parallel to the line joining the points (-1, 5) and (1, 9).*

7.1.3 Intersection of Two Lines

The coordinates of a intersection of two lines are found by solving their simultaneous equations.

Exercise 7.1.11

1. *Find the equation of the straight line that contains the point of intersection of the lines* $3x + 2y = 12$ *and* $5x - y = 7$ *and:*

   (a) *passes through the point (-4, -5).*

   (b) *is parallel the the line* $2x - y + 4 = 0$.

   (c) *is perpendicular to line* $y = 5$.

2. *Find the equation of the line that contains the point of intersection of* $2x + 5y - 19 = 0$ *and* $3x - 4y + 6 = 0$ *and is parallel to the line* $4x - y = 8$.

Exercise 7.1.12

1. *Find the perpendicular bisector of the line joining the points* $(-1,\ 4)$ *and* $(3,\ -2)$.

2. *Show that the point* $(7,\ 5)$ *lies on the perpendicular bisector.*

3. *What kind of triangle is that formed by the points* $(-1,\ 4)$ *,* $(3,\ -2)$ *and* $(7,\ 5)$*?*

Exercise 7.1.13 If AB is the diameter of a circle and P another point on the circumference then Euclidean geometry tells us that $\angle APB = 90°$. **Use this fact to show that the equation of the circle whose diameter has endpoints** $A(x_1,\ y_1)$ **and** $B(x_2,\ y_2)$ **is** $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.