Year 11 Math Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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This edition was printed on February 7, 2017 with worked solutions.
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6 Year 11 Topic 6 — Sequences and Series Part 2

6.1 Logarithms

6.1.1 Base 10 Logarithms

- If a positive number y is written as a power of 10 in the form $y = 10^x$, the index part x is called the logarithm of the number y.
  - This is written as $x = \log_{10} y$.
  - Thus $y = 10^x$ is equivalent to $x = \log_{10} y$.
- Logarithms based on the power of 10 are often called "common logarithms".

6.1.2 The General Definition of a Logarithm

- If a number y is expressed in the form $y = a^x$, then the index part of x is called the logarithm of y to the base a
  - This is written as $x = \log_a y$.
  - Thus $y = a^x$ is equivalent to $x = \log_a y$.

Exercise 6.1.1 Find the value of the following:

1. $\log_2 32$

2. $\log_{\sqrt{5}} 125$

3. $\log_4 64$

4. $\log_2 \frac{1}{4}$

6.1.3 Change of Base Law

- The calculator only evaluates common logarithms.
- This law allows us to change a logarithm from one base to another (i.e. base 10).
- $log_a\, x = \frac{log_{10}\, x}{log_{10}\, a}$

Exercise 6.1.2 Use the change of base law to evaluate the following (correct to 3 decimal places):

1. $\log_4 9$

2. $\log_5 60$

3. $\log_9 56$

4. $\log_5 20$

5. $\log_2 10$

6. $\log_3 40$

7. $\log_2 \frac{1}{2}$

6.1.4 The Remaining Log Laws

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
- $\log_a x^n = n \log_a x$
- $\log_a 1 = 0$
- $\log_a a = 1$

Exercise 6.1.3

1. $\log_5 10 - \log_5 2$

2. $\log_2 \frac{1}{4} - \log_2 \frac{1}{8}$

3. $\log_6 4 + \log_6 9$

4. $\log_3 8 - \log_3 24$

5. $\log_2 32 + \log_2 \frac{1}{2}$

6. $\log_4 \sqrt{16} - \log_4 64$

Exercise 6.1.4 Solve for x in the following:

1. $\log_a x = \log_a 3 + \log_a 5$

2. $\log_a x + \log_a 4 = \log_2 0$

3. $\log_a 6 - \log_a x = \log_a 12$

4. $\log_a(x+1) - \log_a 5 = \log_a x$

5. $\log_x \frac{1}{3} = -\frac{1}{3}$

6.1.5 Miscellaneous Problems

Exercise 6.1.5 Simplify the following:

1. $\frac{\log_a x^{10}}{\log_a x}$

2. $\frac{\log_a 2x^3 - \log_a x^2}{\log_a 2x}$

3. $\frac{\log_a (\frac{x^3}{y}) + \log_a (\frac{y}{x})}{\log_a \sqrt{x}}$

4. $\frac{\log_a 4x + \log_a (\frac{2}{x})}{\log_a 2}$

Exercise 6.1.6 Solve the following equations:

1. $\log_3(x+2) + 2\log_3 9 = 3$

2. $\log_a(x+2) = \frac{1}{3}\log_a 8 + \log_a 3$

3. $3\log_a 5x + \frac{1}{2}\log_a 25 = \log_a 5000$

4. $\log_2 x + \log_2(x^2-4) - \log_2(x+2) = 3$

5. $\frac{\log_2 9}{\log_2 3} = \log_2 2x$

Exercise 6.1.7 Solve each pair of simultaneous equations by converting both sides to a common base:

1. $2^{2x-y} = 32$
   $2^{4x+y} = 128$

2. $5^{x+y} = \frac{1}{5}$
   $5^{3x+2y} = 1$

3. $3^{x+y} = 81$
   $81^{x-y} = 3$

4. $7^{x+y} = 49$
   $49^{x-y} = 7$

Exercise 6.1.8 Practical Exam Questions

1. *If* $\log_{64} a = \frac{1}{3}$, *find the value of a.*

2. *Simplify* $(\log_6 4 + \log_6 9)^2$.

3. *Solve* $\log(x-2) + \log(x-1) = \log(x+2)$.

4. *Simplify* $\log_4 9 + \log_4 8 - 2\log_4 6$.

5. *If* $f(x) = \log(\frac{1+x}{1-x})$, *prove that* $f(u) = 2f(x)$, *where* $u = \frac{2x}{1+x^2}$.