Year 11 Math Homework

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

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6 Year 11 Topic 6 — Sequences and Series Part 1

6.1 Sequences and Series

6.1.1 Indices

Index Laws:

- $a^{\frac{1}{2}} = \sqrt{a}$
- $a^{-1} = \frac{1}{a}$
- $a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}}$
- $a^x \times a^y = a^{x+y}$
- $a^x \div a^y = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$
- $(\frac{a}{b})^x = \frac{a^x}{b^x}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Negative and Zero Bases:

- $(-16)^{\frac{1}{2}}$ is undefined.
- $0^x = 0$ if $x > 0$ but 0^{-2} is undefined.
- 0^0 is undefined.

Exercise 6.1.1 Simplify the following:

1. $\frac{1}{12^{-2}}$ __________
2. $\frac{3^{-2}}{4^{-3}}$ __________
3. $\frac{2^{-2}}{15^2}$ __________

Exercise 6.1.2 Simplify the following, giving the answers without negative indices:

1. $x^{-3}y^{-2} \times x^5 y^3$ __________
2. $3x^{-3} \times 8x^2$ __________
3. $(2x^2y^{-3})^{-2}$ __________
4. $7x^{\frac{3}{4}} \times 3x^{\frac{2}{3}}$ __________
5. $(8x^6y^{-3})^{\frac{1}{3}}$ __________
6. $(\frac{8}{125})^{-\frac{2}{3}}$ __________
7. $(3\frac{3}{8})^{-\frac{4}{3}}$ __________
8. $(0.36)^{-0.5}$ __________

Exercise 6.1.3 Write the following expressions using fractional and negative indices:

1. $\sqrt[3]{5}$ ____________________

2. $\frac{\sqrt[3]{x^4}}{7}$ ____________________

3. $\frac{5}{2x+3}$ ____________________

4. $\frac{2\sqrt{x}}{x}$ ____________________

5. $x\sqrt[3]{x^2}$ ____________________

Exercise 6.1.4 Write as a single fraction without negative indices and fully simplify:

1. $x^{-1} - y^{-1}$

2. $\frac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}$

3. $x^{-2}y^{-2}(x^2y^{-1} - x^{-1}y^2)$

4. $\frac{(x^2-1)^{-1}}{(x-1)^{-1}}$

5. $(x^{-2} - y^{-2})^{-2}$

Exercise 6.1.5 Explain why $3^n + 3^{n+1} = 3^n(1+3) = 4 \times 3^n$. Use similar methods to then simplify the following:

1. $5^{n+3} + 5^n$

2. $2^{2n+2} - 2^{2n-1}$

3. $\frac{2^{2n}-2^{n-1}}{2^n-2^{-1}}$

4. $\frac{7^{n+3}-7^n}{7^n}$

Exercise 6.1.6 Challenge Exercise:

1. *Find the smallest positive integers m and n for which* $12 < 2^{\frac{m}{n}} < 13$

2. *How many factors does the number 2856 have?*

6.1.2 Logarithms

Definition of Logarithms:

- The logarithmic function $y = \log_a x$ is simply the inverse function of the exponential function $y = a^x$
- $\log_a x$ is the index, when x is written as a power of a.
- In symbols, $y = \log_a x$ means $x = a^y$

Laws for Logarithms:

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a \frac{1}{a} = -1$
- $\log_a \sqrt{a} = \frac{1}{2}$
- $\log_a xy = \log_a x + \log_a y$
- $\log_a \frac{x}{y} = \log_a x - \log_a y$
- $\log_a x^n = n \log_a x$
- Change of Base Law: $\log_b x = \frac{\log_a x}{\log_a b}$

Exercise 6.1.7 Rewrite each equation in index form and then solve:

1. $x = \log_2 16$

2. $x = \log_3 \sqrt{3}$

3. $x = \log_{\frac{1}{3}} \frac{1}{81}$

4. $x = \log_5 \frac{1}{\sqrt{5}}$

Exercise 6.1.8 Rewrite each equation in index form and then solve:

1. $\log_7 x = \frac{1}{2}$

2. $\log_8 x = -\frac{2}{3}$

3. $\log_x 3 = \frac{1}{2}$

4. $\log_x 27 = -\frac{1}{3}$

5. $\log_x \frac{4}{9} = 2$

Exercise 6.1.9 Express the following in terms of $\log_2 3$ and $\log_2 5$:

1. $\log_2 18$

2. $\log_2 \frac{1}{6}$

3. $\log_2 2\frac{1}{2}$

4. $\log_2 15\sqrt{3}$

5. $\log_2 \frac{3}{25}\sqrt{30}$

Exercise 6.1.10 Rewrite these relations in index form without using logarithms:

1. $\log_a(x+y) = \log_a x + \log_a y$

2. $\log_2 x = 4\log_2 y$

Exercise 6.1.11 Rewrite these relations in index form without using logarithms:

1. $\frac{1}{2}\log_2 x = \frac{1}{3}\log_2 y - 1$

2. $2\log_3(2x+1) = 3\log_3(2x-1)$

Exercise 6.1.12 Simplify the following expressions:

1. $3^{-\log_3 2}$

2. $5^{-\log_5 x}$

3. $a^{n\log_a x}$

4. $3^{\frac{\log_3 x}{x}}$

6.1.3 Arithmetic Sequences

- The sequence $a,\ a+d,\ a+2d,\ \ldots a+(n-1)d,\ldots$ is called an arithmetic sequence.
  - a and b are real numbers and n is a natural number.
  - a is the first term and d is the common difference.
- It can be defined by the rule $t_n = a + (n-1)d$.

6.1.4 Arithmetic Series

- S series is the sum of the terms of a sequence. thus $t_1 + t_2 + t_3 \ldots + t_n$
- the finite arithmetic series is defined by the quadratic function: $S_n = \frac{n}{2}[2a+(n-1)d]$, $(n \epsilon N)$

Exercise 6.1.13

1. *Find the eleventh term of the sequence 8, 14, 20, . . .*

2. *Find the sum of the first twenty terms of the sequence 4, 7, 10, . . .*

3. *How many terms of the series 3 + 7 + 11 + . . . must be taken to give a sum of 820?*

4. *Find the arithmetic sequence whose series is defined by* $S_n = 2n^2 + n$.

6.1.5 Sigma Notation

Definition: The symbol Σ is used in mathematics to denote "the sum of"

$$\sum_{n=1}^{n} x^n = x + x^2 + x^3 + ... + x^n$$

each of the form x^n and we give n the values $1, 2, 3...n$

6.1.6 Geometric Sequences

- The sequence $a, ar, ar^2, ar^3, ..., ar^{n-1}, ..$is called a geometric sequence.
  - Where a and r are real numbers and n is a natural number.
- It can be defined by the rule: $t_n = ar^{n-1}$

6.1.7 Finite Geometric Series

- The first n terms of the series is denoted by S_n.

$$S_n = a + ar + ar^2 + .. + ar^{n-1} = \sum_{k=1}^{n} ar^{k-1}$$

- Thus the finite geometric series is defined by:

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad or\,, \quad \frac{a(r^n - 1)}{r - 1}$$

Exercise 6.1.14

1. *Find the 8th term and the sum of the first 8 terms of the sequence* $4, 6, 9, ..$

2. *How many terms of the series* $6 + 3 + \frac{3}{2} + ...$ *must be taken to give a sum of* $11\frac{13}{16}$*?*

Exercise 6.1.15 Miscellaneous Exercises (Practical Exam Questions)

1. *Evaluate* $\sqrt{\frac{275.4}{5.2\times3.9}}$ *correct to two significant figures.*

2. *Factorise* $x^3 - 27$.

3. *Express* $\frac{2x-3}{2} - \frac{x-1}{5}$ *as a single fraction in its simplest form.*

4. *Find the value of x for which* $|x - 3| \leq 1$.

5. *Evaluate:*

$$\sum_{n=3}^{5}(2n + 1)$$

6. *Evaluate:*

$$\sum_{k=1}^{4}(-1)^k k^2$$