Year 11 Math Homework

Student Name: ______________	**Grade**: ______________
Date: ______________	**Score**: ______________

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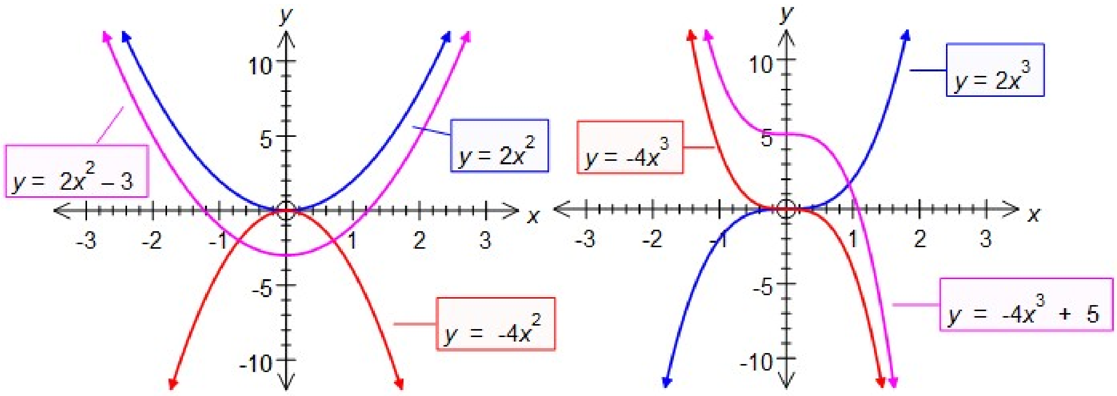
5 Year 11 Topic 5 —- Graphs and Inequations

5.1 Curve Sketching

A sketch is an approximation of the graph which intends to show the main features of the graph.

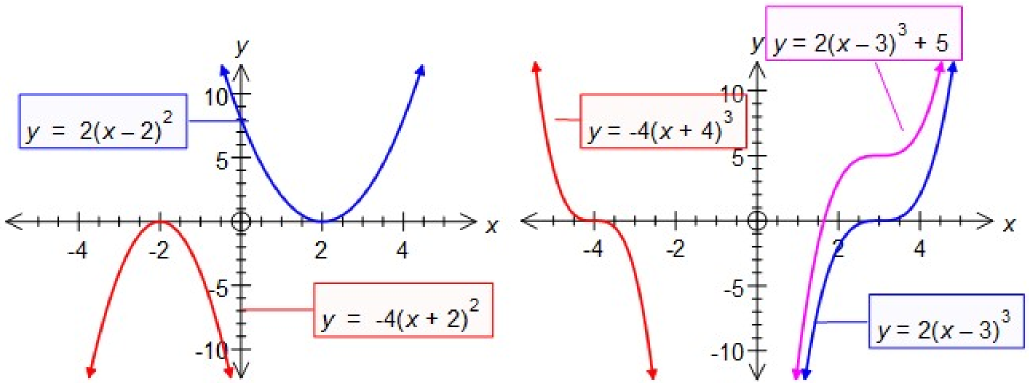
5.1.1 Curves of the Form $y = ax^n$ and $y = ax^n + d$

Definition: Curves with an even value of n will be shaped like a parabola $(y = ax^2)$.
Curves with an odd value of n will be shaped like a cubic $(y = ax^3)$.
If d is positive, translate the curve up d units.
If d is negative, translate the curve down d units.



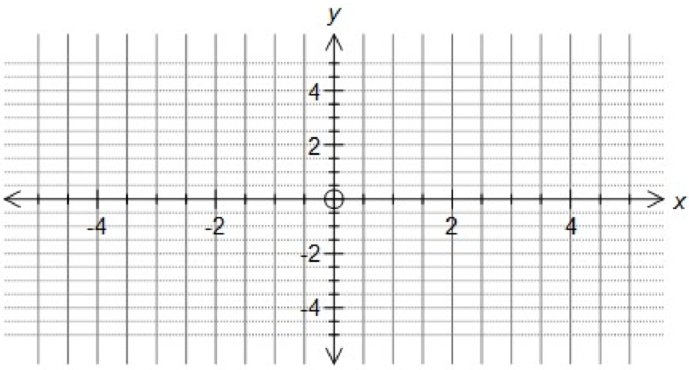
5.1.2 Curves of the Form $y = a(x - r)^n$ and $y = a(x - r)^n + d$

Definition: If the curve $y = ax^n$ is moved horizontally r units to the right,
the equation of the new curve is $y = a(x - r)^n$.
If the curve $y = ax^n$ is moved horizontally r units to the left,
the equation of the new curve is $y = a(x + r)^n$.
If d is positive, translate the curve up d units.
If d is negative, translate the curve down d units.

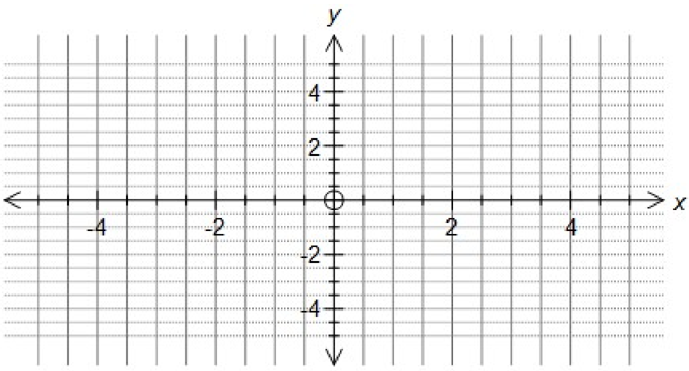


Exercise 5.1.1 Make sketches of the following curves:

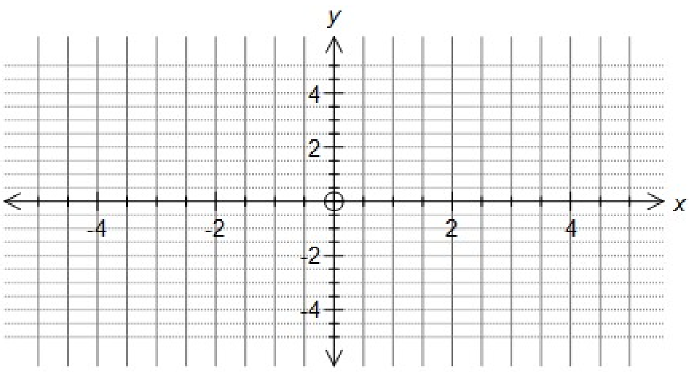
1. $y = 2(x-2)^2 + 2$



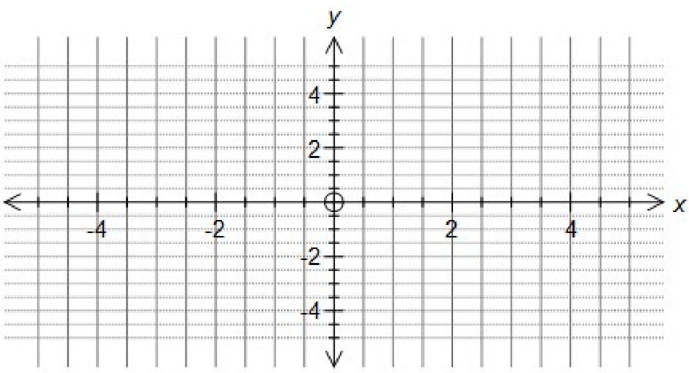
2. $y = 3(x-3)^3 + 3$



3. $y = (x+1)^4 - 1$



4. $y = -2(x+3)^5 - 2$



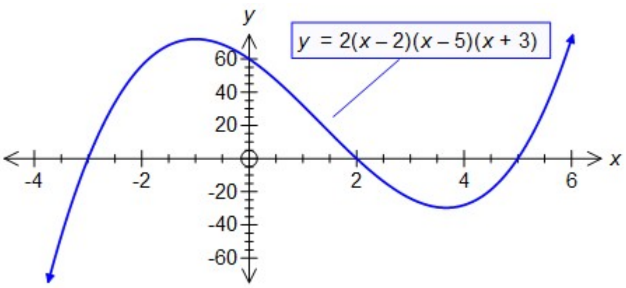
5.1.3 Curves of the Form $y = a(x-r)(x-s)(x-t)$

Definition: To sketch the curve $y = a(x-r)(x-s)(x-t)$:
Find the x-intercepts by putting $y = 0$
Test a point in each of the sections.

Example 5.1.1 Make sketches of the curve $y = 2(x-2)(x-5)(x+3)$

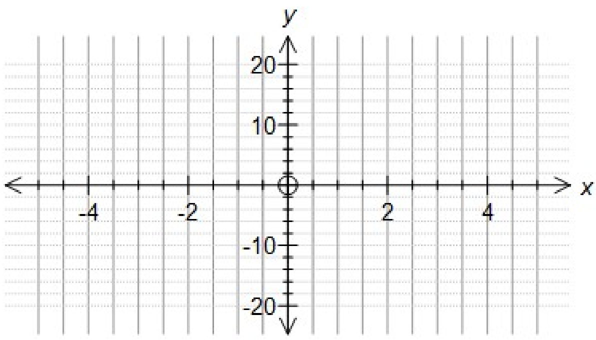
Solution: $y = 2(x-2)(x-5)(x+3)$, *Let* $y = 0 \Rightarrow (x-2)(x-5)(x+3) = 0$
The curve has x-intercepts at $x = 2,\ x = 5$ *and* $x = -3$.
Testing values of x shows that the curve is in different regions.

x	-4	-3	0	2	3	5	6
y	(-)	0	(+)	0	(-)	0	(+)

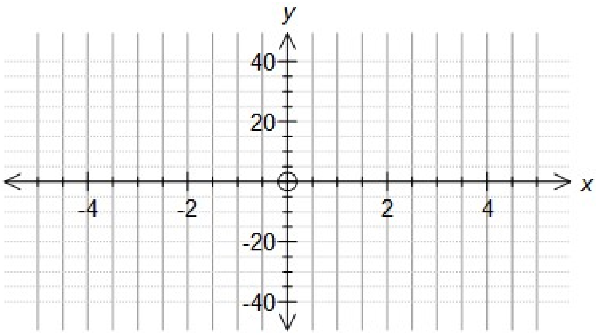


Exercise 5.1.2 Make sketches of the following curves:

1. $y = -2(x-1)(x-3)(x+2)$

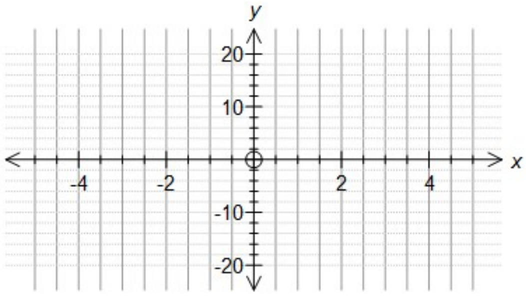


2. $y = (x-2)^2(x+4)$

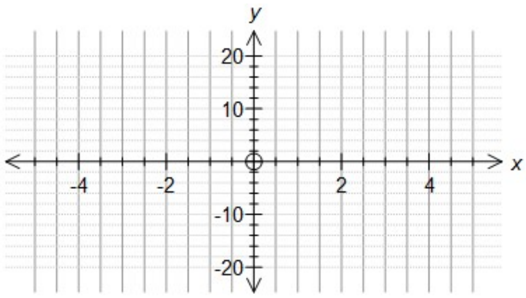

y
40
20
x
-4
-2
2
4
-20
-40

Exercise 5.1.3

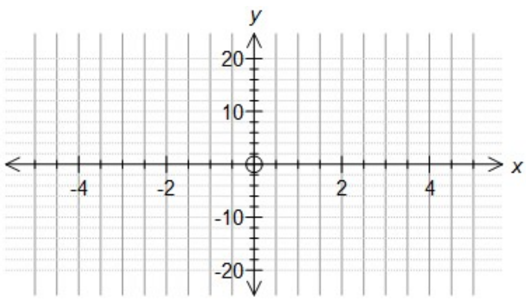
1. *On the same number plane, sketch the curves* $y = -x(x+3)(x-2)$ *and* $y = -2x(x+3)(x-2)$.



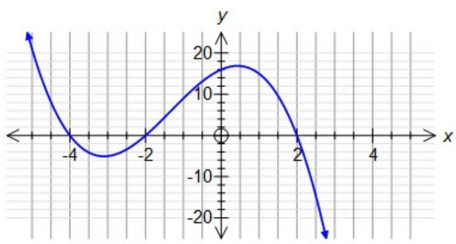
2. *On the same number plane, sketch the curves* $y = (x-2)(x+2)(x-4)$ *and* $y = -(x-2)(x+2)(x-4)$.

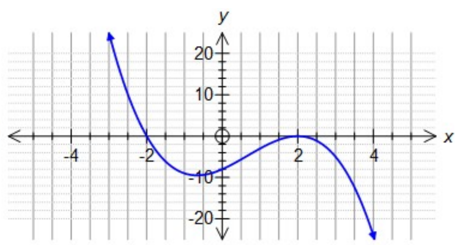


3. *On the same number plane, sketch the curves* $y = 2(x+1)^2(x-3)$ *and* $y = -(x+1)^2(x-3)$.



Exercise 5.1.4 Write down a possible equation for each of the curves shown below:





5.1.4 The Circle and its Equations

Definition: The equation of a circle which has center $(p,\ q)$ and radius r is given by the equation: $(x-p)^2+(y-q)^2=r^2$

Example 5.1.2

1. *What is the equation of a circle that has centre $(-2,4)$ and radius 3?*

Solution: *The circle formula is* $(x-p)^2+(y-q)^2=r^2$, *where* $(p,\ q)$ *is* $(-2,\ 4)$ *and* $r=3$.

$\therefore$ *the equation of the circle is* $(x+2)^2+(y-4)^2=9$.

2. *Find the centre and radius of the circle* $(x-5)^2+(y+7)^2=16$.

Solution: $(x-5)^2+(y+7)^2=16$ *can be written as* $(x-5)^2+(y-(-7))^2=4^2$

$\therefore$ *the circle has centre* $(5,\ -7)$ *and radius* 4.

3. *Find the centre and radius of the circle* $x^2+y^2+8x-6y=0$.

Solution: *We can write the equation in the form* $(x-p)^2+(y-q)^2=r^2$

By completing the square:

$x^2+y^2+8x-6y=0,\ \Rightarrow\ x^2+8x+16+y^2-6y+9=25,$

$(x+4)^2+(y-3)^2=25,\ \Rightarrow\ (x-(-4))^2+(y-3)^2=5^2$

$\therefore$ *the circle has centre* $(-4,\ 3)$ *and radius* 5.

Exercise 5.1.5

1. *Find the centre and the radius of the circle* $x^2+y^2-6x+2y+6=0$

2. *A circle with centre (3, -4) passes through the origin. What is its equation?*

5.2 Graphs and Inequations

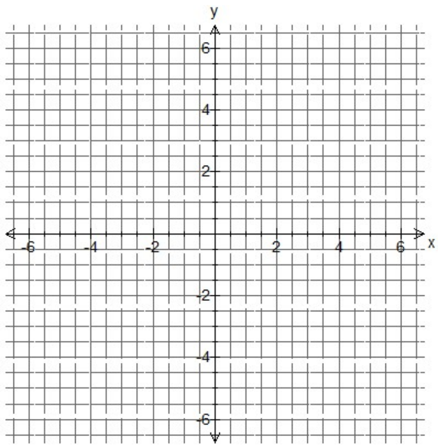
5.2.1 Regions in the Number Plane

Graphing the region corresponding to an inequation:

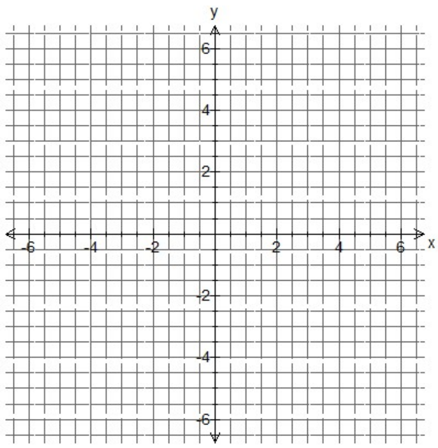
- *The Boundary:* Replace the inequality symbol by an equal symbol and graph the curve. It should be drawn broken if exclusive and unbroken if inclusive.
- *Shading:* Determine which parts are included and excluded, and shade the parts that are included. Take one or more test points not on any boundary and substitute into the LHS and RHS of the original inequation. The origin is the easiest test point or points on the axes.
- *Checking Boundaries and Corners:* Check that boundaries are correctly broken or unbroken. Corner points must be marked clearly with a closed circle if they are included, or an open circle if excluded.

Exercise 5.2.1

1. *Sketch the region* $x^2 + y^2 > 16$.

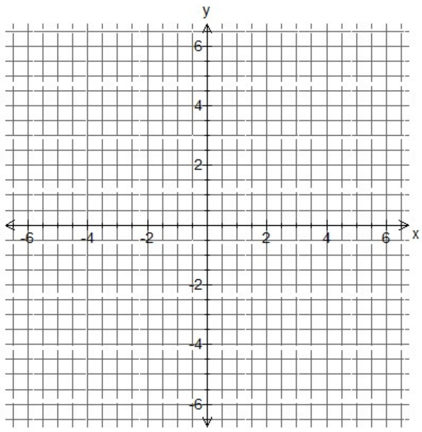


2. *Sketch* $y > x^2$.

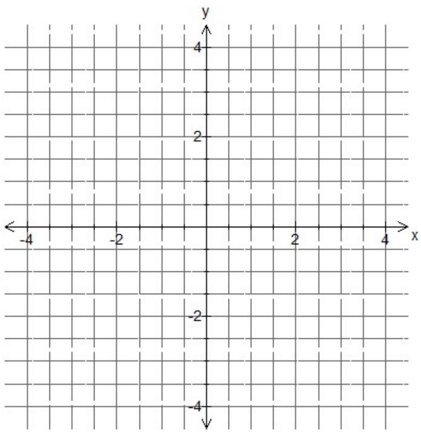


Exercise 5.2.2

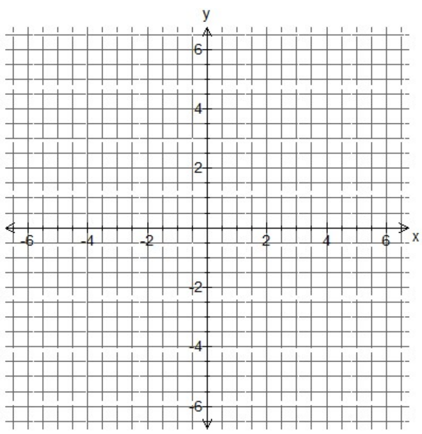
1. *Sketch* $(x-2)^2+(y+1)^2>9$



2. *Sketch* $y \geq x^2-1$.

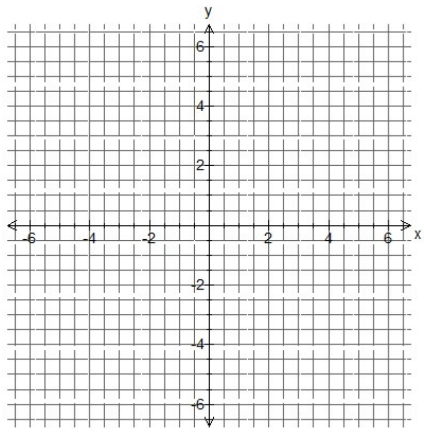


3. *Sketch* $y \leq (2x+1)(x-3)$

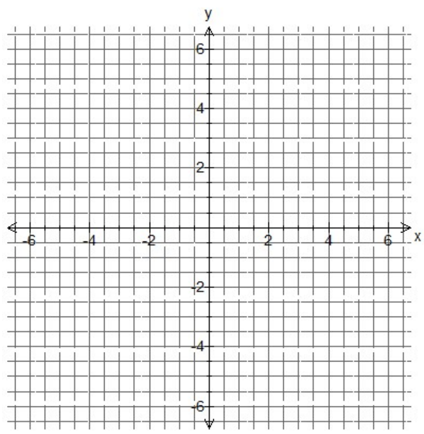


Exercise 5.2.3 For each inequation, sketch the regions:

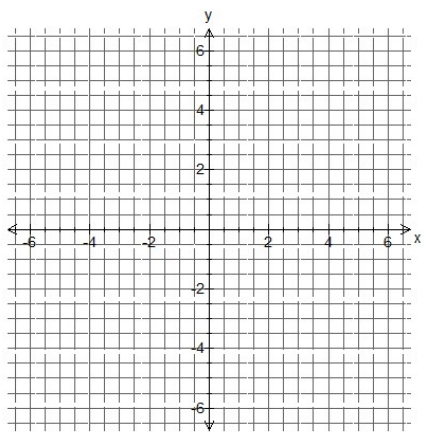
1. $y < x^2 - 2x - 3$



2. $y \geq 2 + x - x^2$

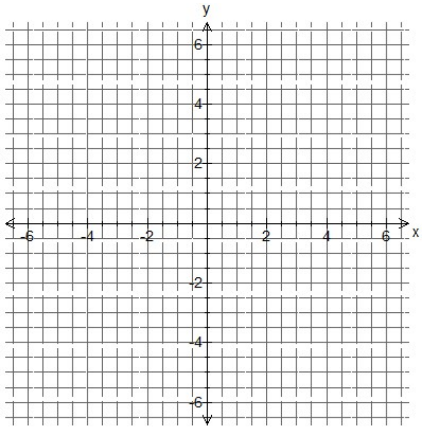


3. $y < (2x - 3)(x + 1)$

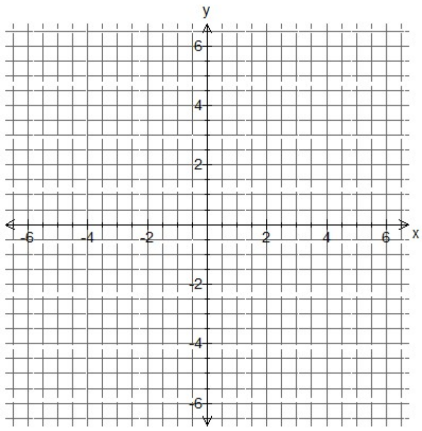


Exercise 5.2.4

1. *Sketch* $x \leq \frac{1}{y}$



2. *Sketch the intersection of* $x^2 + y^2 > 1$ *and* $x^2 + y^2 \leq 9$. *What is the union of these two regions?*



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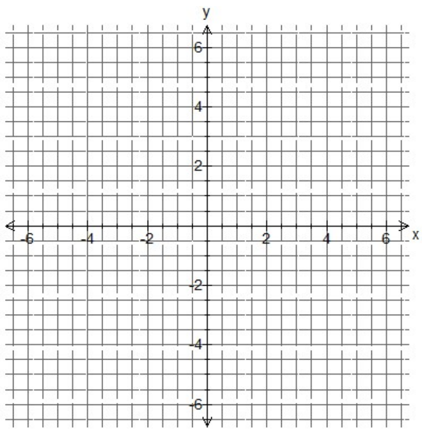
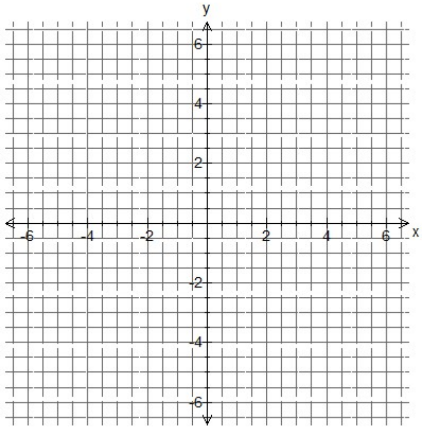
3. *Find the intersection points of the line* $y = 5 - x$ *and the circle* $x^2 + y^2 = 25$.

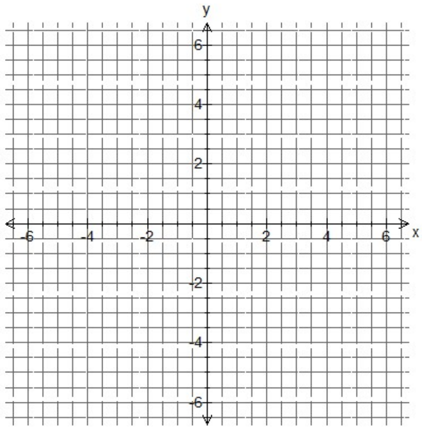


Exercise 5.2.5 Sketch the region indicated by:

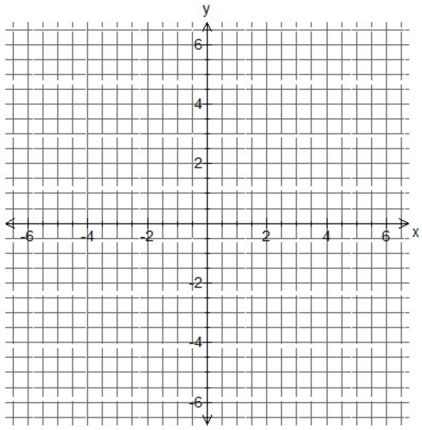
1. $y < x + 1$ *and* $y \geq -\frac{1}{2}x - 2$



2. $y < x + 1$ *and* $y \geq -\frac{1}{2}x - 2$ *and* $y < 4x - 2$

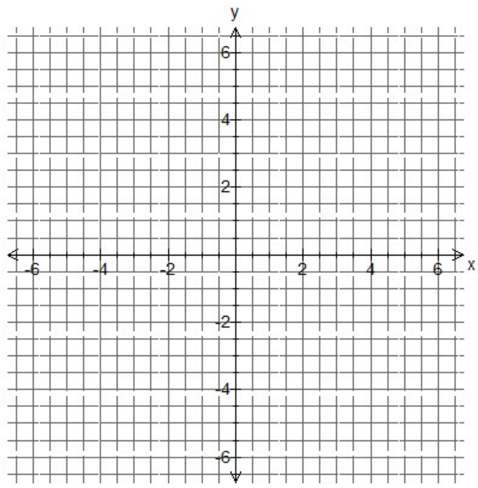


3. $y < x + 1$ *or* $y \geq -\frac{1}{2}x - 2$ *or* $y < 4x - 2$

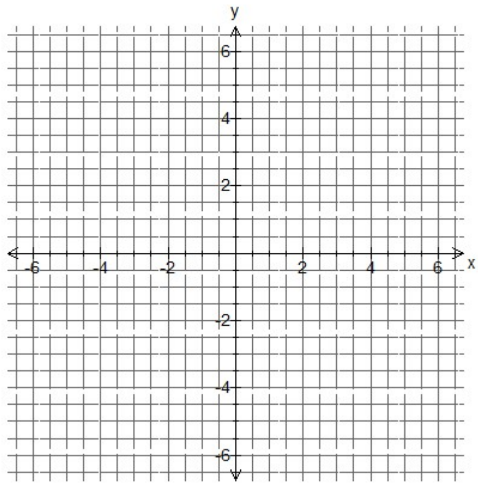


Exercise 5.2.6

1. *The inequation* $|x| < 4$ *implies the intersection of two regions. Write down the equation of these two regions. Hence sketch the region* $|x| < 4$.

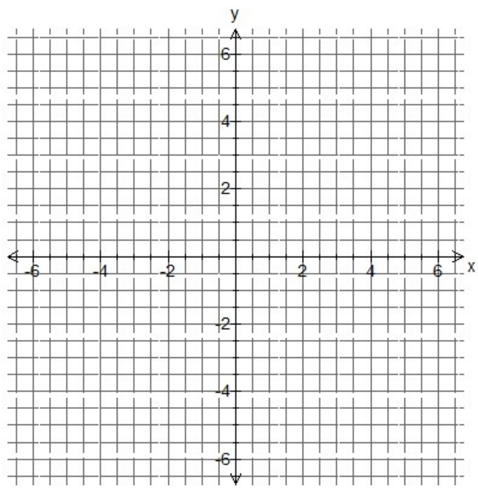


2. *The inequation* $|x - y| \leq 3$ *implies the intersection of two regions. Write down the equation of these two regions. Hence sketch the region* $|x - y| \leq 3$.

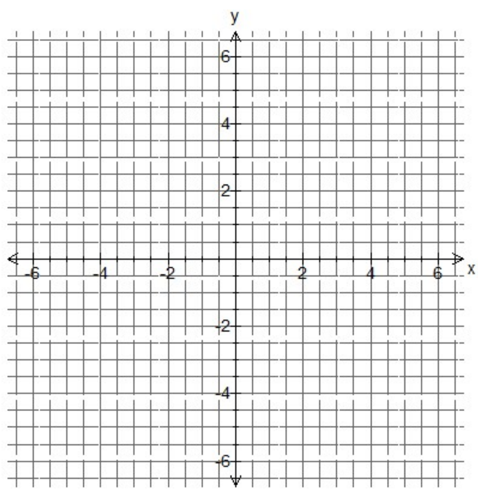


Exercise 5.2.7

1. *The inequation* $|y| \geq 2$ *implies the intersection of two regions. Write down the equation of these two regions. Hence sketch the region* $|y| \geq 2$.

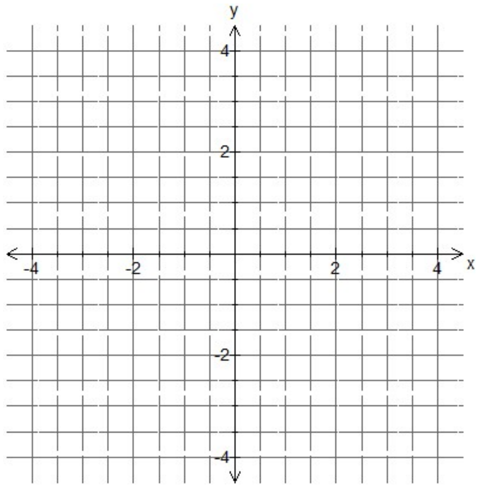


2. *The inequation* $|y + 2x| \geq 2$ *implies the intersection of two regions. Write down the equation of these two regions. Hence sketch the region* $|y + 2x| \geq 2$.

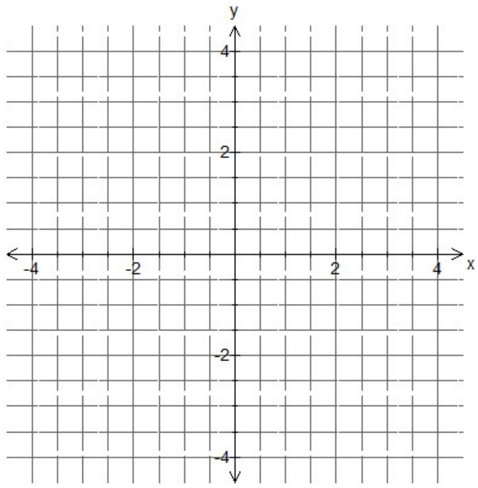


Exercise 5.2.8

1. *Sketch the region* $x^2 + y^2 \geq 4$ *for the domain* $x > -1$ *and range* $y < 2$ *and give the coordinates of each corner.*

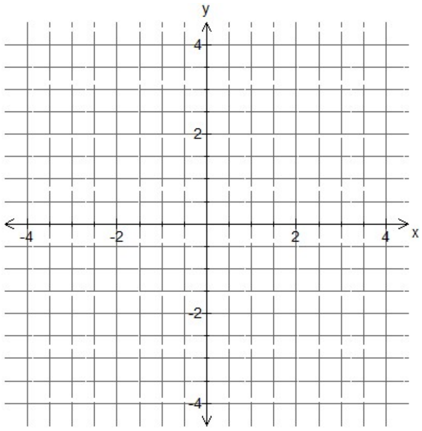


2. *Sketch the region* $y \leq x^2 - 2x + 2$ *with* $y \geq 0$ *and* $0 \leq x \leq 2$ *and give the coordinates of each corner.*

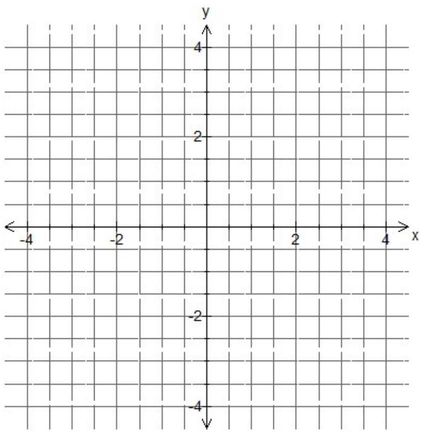


Exercise 5.2.9 Carefully sketch the following regions, paying attention to implied boundaries:

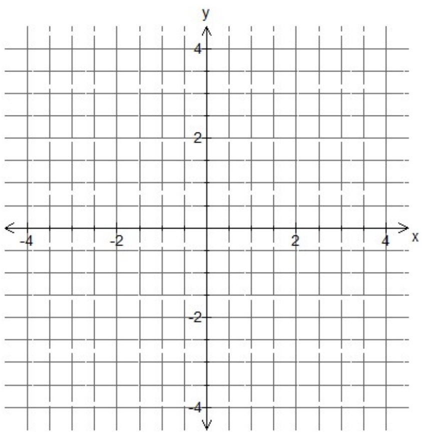
1. $y < \sqrt{4 - x^2}$.



2. $x > \sqrt{9 - y^2}$.

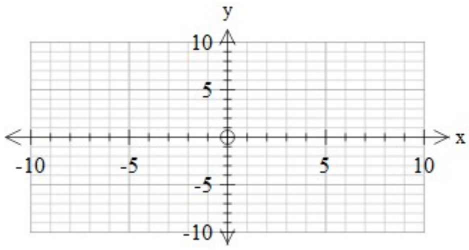


3. $y > \frac{1}{x}$.



Exercise 5.2.10

1. *Find the x-intercept, y intercept and vertex of the parabola* $y = 2(x-1)^2 + 3$*, and hence sketch the parabola.*



2. *The equation* $2(x-1)^2 + 3 = k$*, where* k *is a constant, is to have two roots. Use your graph to find the possible values the k may have.*

Exercise 5.2.11 The roots of $4kx^2 - (3k-4)x + 1 = 0$**. Find the value of k in each case:**

1. *The roots are real.*

2. *The roots are reciprocals.*