Year 11 Math Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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4 Year 11 Topic 4 — Numbers and Functions (Part 2)

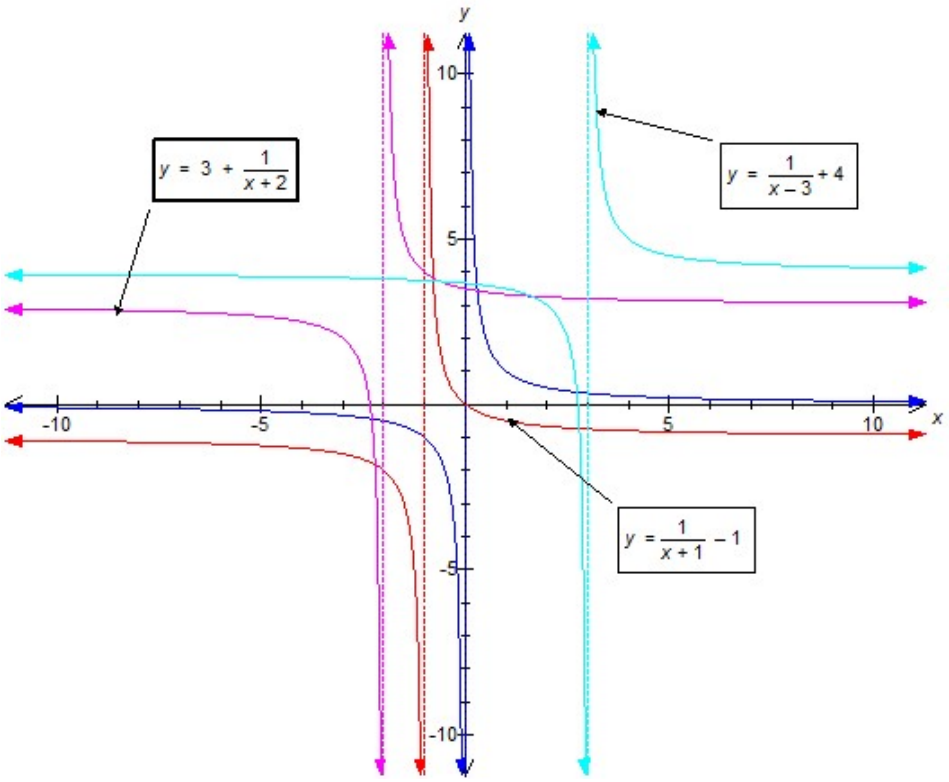
4.1 Numbers and Functions (Revision)

4.1.1 Shifting and Reflection of Known Graphs

- To shift k units to the right, replace x by $x-k$. Alternatively, if the graph is a function, the new function rule is $y=f(x-k)$.
- To shift l units upwards, replace y by $y-l$. Alternatively, if the graph is a function, the function rule is $y=f(x)+l$.
- To reflect in the y-axis, replace x by $-x$. Alternatively, if the graph is a function, the function rule is $y=f(-x)$.
- To reflect in the x-axis, replace by y by $-y$. Alternatively, if the graph is a function, the function rule is $y=-f(x)$.
- To reflect in the line $y=x$, replace x by y and y by x.

Example 4.1.1 Sketch $y=\frac{1}{x}$, then use the shifting procedures to sketch the following graphs. Find any x-intercepts and y-intercepts, and mark them on your graph:

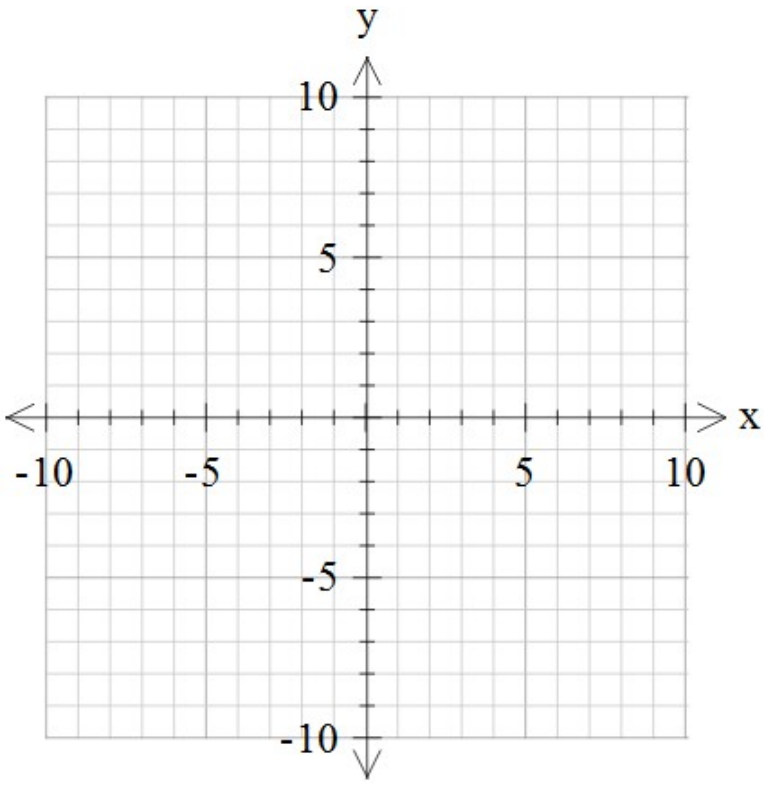
(1). $y=\frac{1}{x+1}-1$ *(2).* $y=3+\frac{1}{x+2}$ *(3).* $y=\frac{1}{x-3}+4$



Exercise 4.1.1 Complete the square, then sketch each of these circles, stating the centre and radius. By substituting $x = 0$ and $y = 0$, find any intercepts with the axes:

1. $x^2 - 4x + y^2 - 10y = -20$

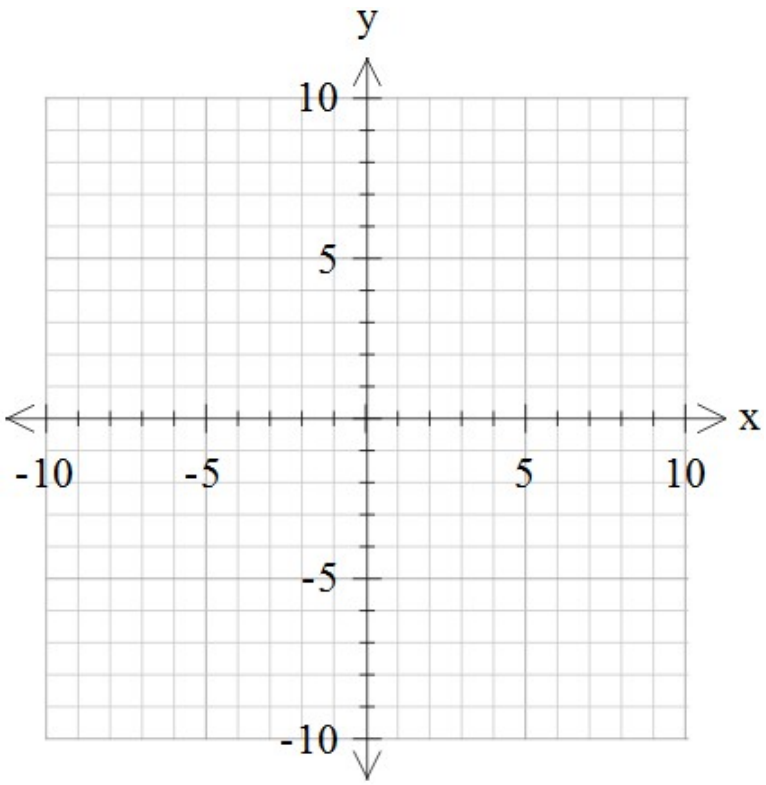
2. $x^2 - 2x + y^2 + 4y = 1$



Exercise 4.1.2

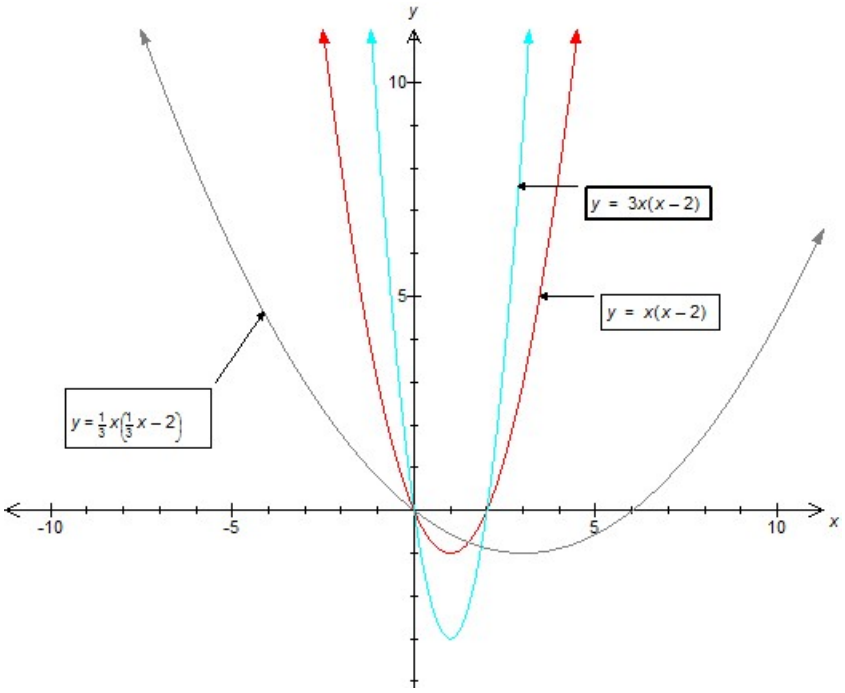
1. *Find the equation of the circle* $(x-2)^2+(y+1)^2=25$ *after it has been reflected in the line* $y=x$.

2. *Find the coordinates of the points where the two circles intersect.*

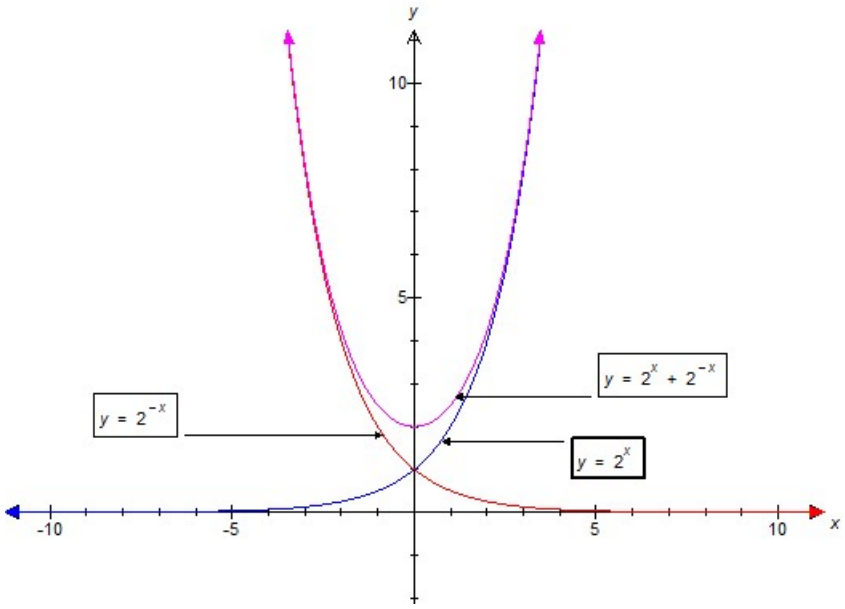


4.1.2 Further Transformations of Known Graphs

- To stretch the graph in a vertical direction by a factor of a, replace x by a. Alternatively, if the graph is a function, the new function rules is $y = a\,f(x)$
- To stretch the graph in a horizontal direction by a factor of a, replace x by $\frac{x}{a}$. Alternatively, if the graph is a function, the new function rule is $y = f(\frac{x}{a})$



- The graph of the sum or difference of two functions can be obtained from the graphs of two functions by adding or subtracting the heights at each value of x.

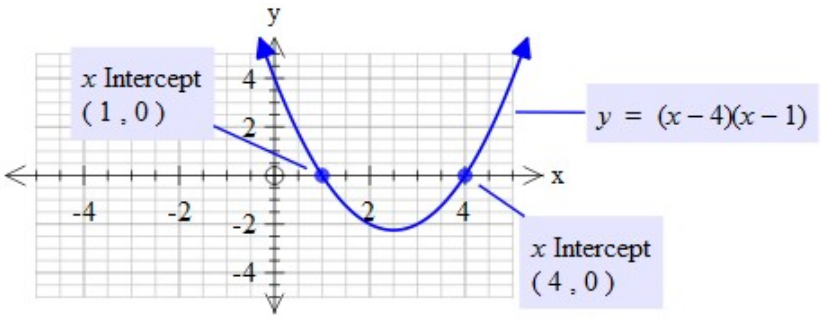


4.2 Graphs and Inequations

4.2.1 Inequations

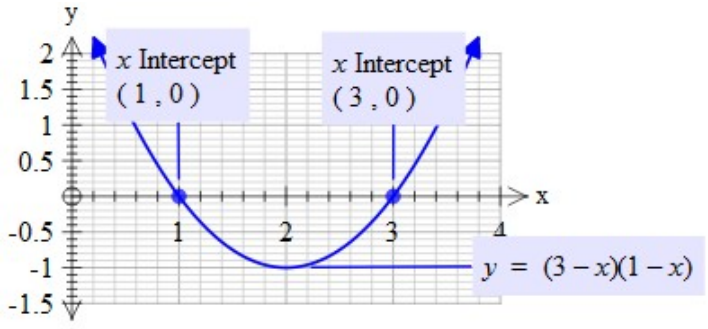
- *Linear Inequations:* When multiplying or dividing both sides of an inequation by a negative, the inequality symbol is reversed.
- *Quadratic Inequations:* To solve a quadratic inequation, move everything to the LHS, sketch the graph of the LHS, showing the x-intercepts, then read the solution off the graph.
- *Variable in the denominator:* Multiply through by the square of the denominator, being careful to exclude the zeroes of the denominator.

Example 4.2.1 Factorise the LHS and draw an appropriate parabola, hence solving $x^2-5x+4 \geq 0$



Solution: $x^2 - 5x + 4 \geq 0 \;\Rightarrow\; (x-4)(x-1) \geq 0 \; x \geq 4 \, and \, x \leq 1.$

Example 4.2.2 Multiply through by the square of the denominator and solve $\frac{2}{3-x} > 1$**.**

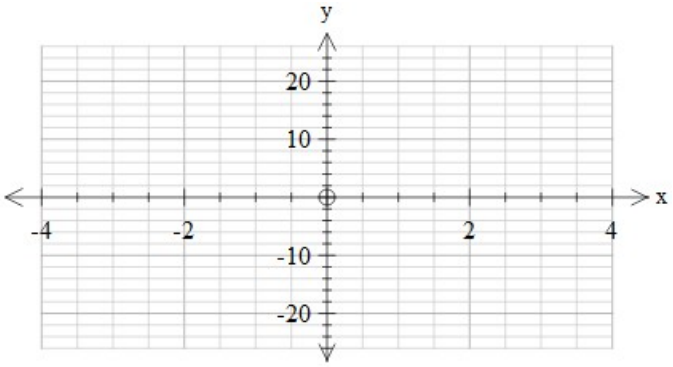


Solution: $\dfrac{2}{3-x} \times (3-x)^2 > (3-x)^2 \;\Rightarrow\; 2(3-x) > 9 - 6x + x^2 \;\Rightarrow\; 0 > 3 - 4x + x^2$

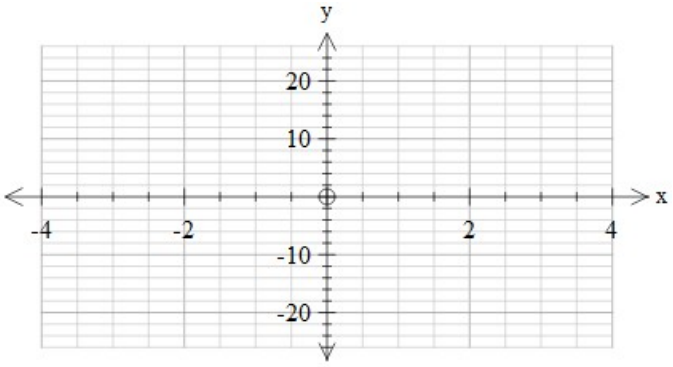
$(3-x)(1-x) < 0 \;\therefore\; 1 < x < 3 \; (x \neq 3).$

Exercise 4.2.1 Factorise the LHS and draw an appropriate parabola in order to solve:

1. $2x^2 - x - 3 \leq 0$



2. $4 + 3x - x^2 > 0$



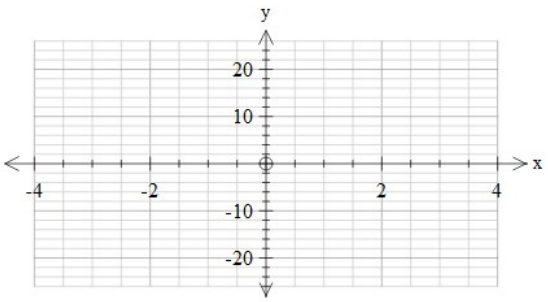
Exercise 4.2.2 Consider the inequation $|x + \frac{1}{x}| < 2x$.

1. *Explain why x must be positive.*

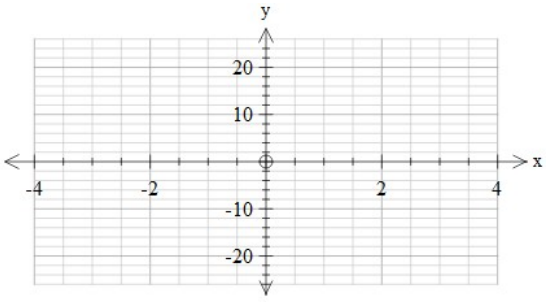
2. *Hence solve the inequation.*

Exercise 4.2.3 Multiply through by the square of the denominator and hence solve:

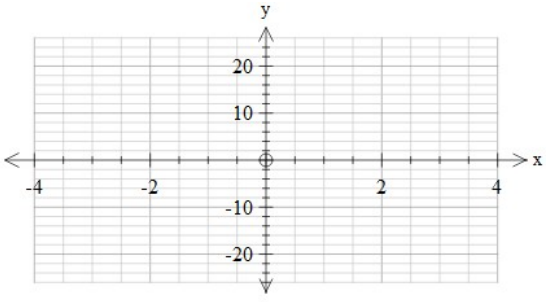
1. $\frac{5}{2x-3} < 3$



2. $\frac{5x}{2x-1} \geq 3$



3. $\frac{4x+7}{x-2} > -3$



Exercise 4.2.4 If $-1 \leq t \leq 3$, what is the range of the values of the following:

1. $2t - 3$

2. $\frac{1}{2}(2t - 1)$

3. $\sqrt{t + 2}$

Exercise 4.2.5

1. *What range of values may $x^2 + 3$ take if:*

   (a) $2 < x < 4$

   (b) $-1 < x \leq 3$

2. *Suppose $x > y > 0$.*

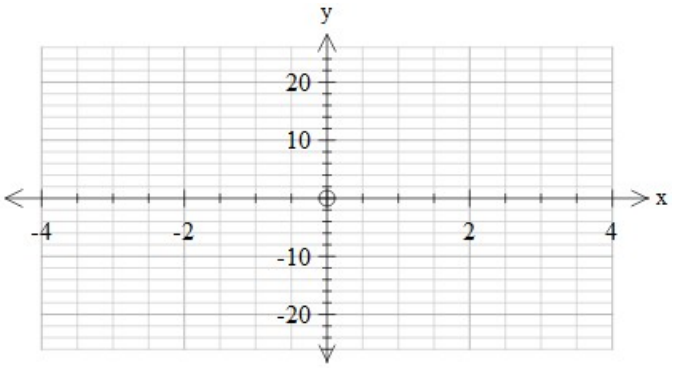
   (a) *Show that $x^2 > y^2$*

   (b) *For what values of n is $x^n > y^n$*

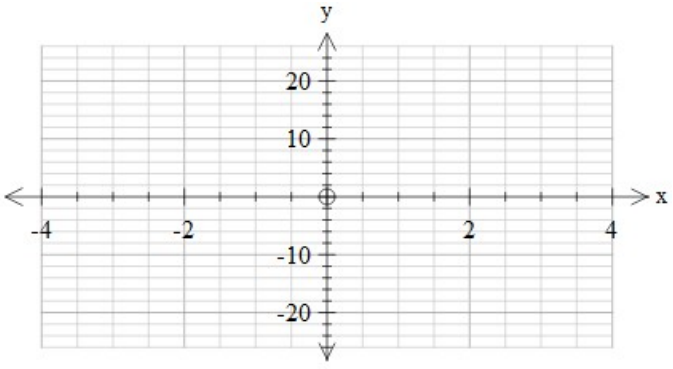
4.2.2 Intercepts and Signs

Exercise 4.2.6 Find all zeroes of these functions, any values of x where the function is discontinuous. Then analyse the sign of the function by taking test points around these zeroes and discontinuities:

1. $f(x) = \frac{x}{x-3}$

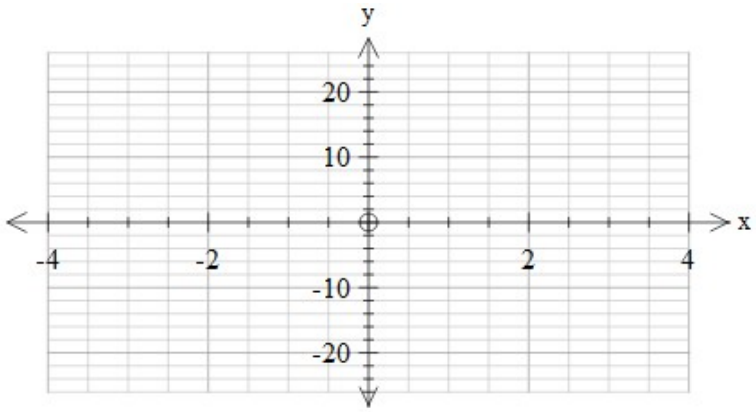


2. $f(x) = \frac{x+3}{x+1}$

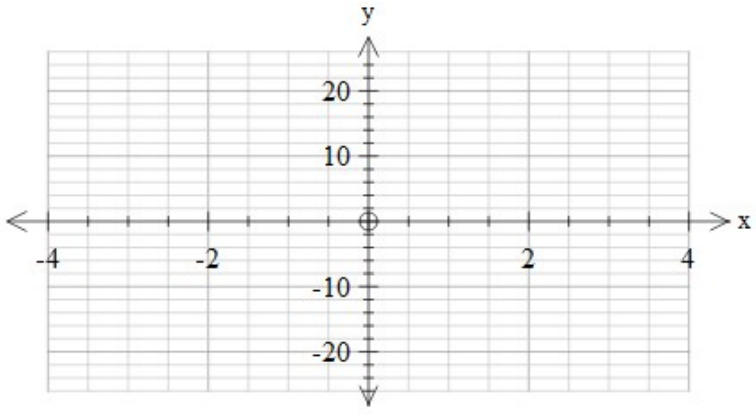


Exercise 4.2.7 Multiply through by the square of the denominator. Collect all terms on one side and then factor to obtain a factored cubic. Sketch this cubic by examining the intercepts and the sign. Hence solve the original inequality:

1. $\frac{4}{x+3} \geq x$



2. $\frac{8}{2x-3} \leq 2x - 1$



4.2.3 The Absolute Value Function

Suppose that $a \geq 0$ and $f(x)$ is a function of x:

- To solve $|f(x)| = a$, write $f(x) = a$ or $f(x) = -a$
- To solve $|f(x)| < a$, write $-a < f(x) < a$
- To solve $|f(x)| > a$, write $f(x) > a$ or $f(x) < -a$

Exercise 4.2.8 Solve the following equations:

1. $|x - 3| = 7$

2. $|3x + 2| = 8$

Exercise 4.2.9 Solve the following inequations:

1. $|x - 2| < 3$

2. $|3x - 5| \leq 2$

3. $|5x + 4| \geq 6$

Exercise 4.2.10

1. *Solve the following double inequations:*

   (a) $2 < |x + 4| < 6$

   (b) $1 \leq |2x - 5| < 4$

2. *Consider the function* $y = \frac{1}{|x-1|}$.

   (a) What is its natural domain?

   (b) Write down the equations of the two branches of the function and sketch its graph.