Year 11 Math Homework

Student Name: ______________	**Grade**: ______________
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3 Year 11 Topic 3 — Basic Algebra Part 3

3.1 Equations and Inequations

3.1.1 Square Roots and Absolute Values

Exercise 3.1.1 Write expressions for the following:

1. $\sqrt{x^2} + x$

2. $|x-3| + |x+3|$

3. $|\sqrt{3} - 3|$

Exercise 3.1.2 Solve for x:

1. $|x-3| = 2$

2. $|3x-5| = 2$

3. $|2x+1| = 2$

Exercise 3.1.3 Solve the following for x:

1. $|2x-1| = |x+4|$

2. $|5x+2| = |4x+7|$

3. $|2x+5| = 3x+9$

3.1.2 Inequations involving Absolute Values

- If $|x| = 5$, then x is 5 units from the origin 0; i.e. $x = \pm 5$.
- If $|x| < 5$, then x is less than 5 units from the origin 0; i.e. $-5 < x < 5$.
- If $|x| > 5$, then x is more than 5 units from origin 0; i.e. $x > 5$ or $x < -5$

Exercise 3.1.4 Solve the following inequations:

1. $|2x+5| > 3$

2. $|6x+5| \leq 1$

3. $|\frac{3x-1}{2}| > 1$

3.1.3 Simultaneous Equations

Exercise 3.1.5

1. $2x + 3y = 14$
   $4x - 5y = 17$

2. $x + \frac{3y}{4} = -1$
   $2x - \frac{y}{4} = 5$

3. $\frac{2x+1}{3} = \frac{y-3}{2}$
   $\frac{2x+1}{2} + 1 = \frac{3y-1}{5}$

4. $3(x - y) - 8(x + y) = 7$
   $2(x + y) + 5(x - y) = -65$

3.1.4 Problem Solving using Simultaneous Equations

Exercise 3.1.6

1. *There are 760 students at a particular high school. If there are 60 more boys than girls, how many boys and girls are there?*

2. *A box contains 100 coins made up of 10-cent coins and 20-cent coins. How many of each kind are there in the box if the value of the coins is $14?*

3. *Find the equation of the straight line that contains the points:*

   (a) *(0, 4) and (1, 0)*

   (b) *(4, 6) and (-6, 8)*

3.1.5 Simultaneous Equations - One Linear, One Second Degree

Exercise 3.1.7

1. $y = 5x + 6$
   $y = x^2$

2. $x + y = 15$
   $y = x^2 - 6x + 1$

3. $3x + y = 11$
   $2x^2 - xy - y = 10$

4. $x + y = 2$
   $x^2 + y^2 = 2$

3.2 Number Systems

3.2.1 Decimal Numbers

You are familiar with the decimal number systems because you use decimal numbers every day. Although decimal numbers are commonplace, their weighted structure is often not understood.

- The decimal number system has ten digits. (0, 1, 2, . . . 9)
- The decimal number system has a base of 10.
  - For example: $25 = 2 \times 10 + 5 \times 1$.
  - The digit 2 has a weight of 10 in this position.
  - The digit 5 has a weight of 1 in this position.
  - The weights for whole numbers are positive powers of ten that increase from right to left, beginning with $10^0 = 1$. $(...10^4 10^3 10^2 10^1 10^0)$
  - For fractional numbers, the weights are negative powers of ten that decrease from left to right beginning with $10^{-1} = 0.1$ $(...10^4 10^3 10^2 10^1 10^0 .10^{-1} 10^{-2} 10^{-3} ...)$
- The value of a digit is determined by its position in the number.

Example 3.2.1 Express the following decimal numbers as a sum of the value of each digit:

1. $48 = 4 \times 10^1 + 8 \times 10^0$
   $= 40 + 8$
2. $325.76 = 3 \times 10^2 + 2 \times 10^1 + 5 \times 10^0 + 7 \times 10^{-1} + 6 \times 10^{-2}$
   $= 300 + 20 + 5 + 0.7 + 0.06$

3.2.2 Binary Numbers

The binary number system is another way to represent quantities.

- It is less complicated than the decimal system because it has only two digits (bits).
- The binary number system has a base of 2.
- The value of a bit is determined by its position in the number.
- In general, with n bits you can count up to a number equal to $2^n - 1$.
- For example, with five bits (n=5) you can count from zero to thirty-one. $(2^5 - 1 = 31)$
- The weight or value of a bit increased from right to left in a binary number.
  $2^4 2^3 2^2 2^1 2^0 .2^{-1} 2^{-2} 2^{-3}$

Example 3.2.2 Binary-to-Decimal Conversion

1. *Convert the binary whole number 1101101 to decimal.*
   Solution: $$\begin{aligned}1101101 &= 2^6 + 2^5 + 2^3 + 2^2 + 2^0\\ &= 64 + 32 + 8 + 4 + 1\\ &= 109\end{aligned}$$

2. *Convert the fractional binary number 0.1011 to decimal.*
   Solution: $$\begin{aligned}0.1011 &= 2^{-1} + 2^{-3} + 2^{-4}\\ &= 0.5 + 0.125 + 0.0625\\ &= 0.6875\end{aligned}$$

Example 3.2.3 Decimal-to-Binary Conversion

1. *Sum-of-Weights Method: Convert the decimal number 58 to binary.*
   Solution: $$\begin{aligned}58 &= 32 + 16 + 8 + 2\\ &= 2^5 + 2^4 + 2^3 + 2^1\\ &= 111010\end{aligned}$$

2. *Repeated Division-by-2 Method: Convert the decimal number 58 to binary.*
   Solution:

Division	*Remainder*	*Binary*
$58 \div 2$	*= 29 with a remainder of 0*	*0 (Least Significant Bit)*
$29 \div 2$	*= 14 with a remainder of 1*	*1*
$14 \div 2$	*= 7 with a remainder of 0*	*0*
$7 \div 2$	*= 3 with a remainder of 1*	*1*
$3 \div 2$	*= 1 with a remainder of 1*	*1*
$1 \div 2$	*= 0 with a remainder of 1*	*1*

So the binary number is 111010.

Exercise 3.2.1 Change the following numbers to binary:

1. *125* ______________________________

2. *256* ______________________________

3. *1004* ______________________________

3.2.3 Hexadecimal Numbers

- The hexadecimal number system has sixteen characters (consists of digits 0-9 and letters A-F).
- Each hexadecimal digit represents a 4-bit binary number.
- *To convert a binary number to hexadecimal:*
  - Break the binary number into 4-bit groups.
  - Start at the right-most bit and replace each 4-bit group with the equivalent hexadecimal symbol.
  - For example: $1100\,1010\,0101\,0111 = CA57_{16}$
- *To convert a hexadecimal number to a binary:*
  - Reverse the process and replace each hexadecimal symbol with the appropriate four bits.
  - For example: $10A4_{16} = 0001\,0000\,1010\,0100$
- *To convert a hexadecimal number to a decimal:*
  - One way to find the decimal equivalent of a hexadecimal number is to first convert the hexadecimal number to binary and then convert from binary to decimal.
  - For example: Convert $1C_{16}$ to decimal.
    Solution: $2C_{16} = 0010\,1100 = 2^5 + 2^3 + 2^2$
    $= 32 + 8 + 4 = 44_{10}$
  - Another way to convert a hexadecimal number to its decimal equivalent is to multiply the decimal value of each hexadecimal digit by its weight and then take the sum of these products.
  - For example: Convert $2C_{16}$ to a decimal number.
    Solution: $2C_{16} = (2 \times 16) + (C \times 1)$
    $= (2 \times 16) + (14 \times 1)$
    $= 32 + 12 = 44$
- *To convert a decimal number to hexadecimal:*
  - Repeated division of a decimal number by 16 will produce the equivalent hexadecimal number, formed by the remainder of the divisions.
  - For example: convert the decimal number 650 to a hexadecimal.
    Solution: $650 = A82_{16}$

Division	Remainder	Hexadecimal
$650 \div 16 = 40.625$	$0.625 \times 16 = 10$	A (Least significant)
$40 \div 16 = 2.5$	$0.5 \times 16 = 8$	8
$2 \div 16 = 0.125$	$0.125 \times 16 = 2$	2 (Most Significant)

3.2.4 Octal Numbers

- The octal number system has a base of 8.
- The octal number system is composed of eight digits. (0, 1, 2, 3, 4, 5, 6, 7)
- *To convert an octal number to decimal:*
  - Multiply each digit by its weight and add their products.
  - Octal digit weight: ...$8^3\,8^2\,8^1\,8^0$
  - For example: convert 2734_8 to a decimal number.
    Solution: $2734_8 = (2 \times 8^3) + (7 \times 8^2) + (3 \times 8^1) + (4 \times 8^0)$.
    $= (2 \times 512) + (7 \times 64) + (3 \times 8) + 4 \times 1)$
    $= 1024 + 192 + 56 + 4$
    $= 1276_{10}$
- *To convert a decimal number to octal:*
  - Repeated division of a decimal number by 8, formed by the remainder of the divisions.
  - For example: convert 259 to an octal number.
    Solution:

Divisions	Remainder	Octal number
$359 \div 8 = 44.875$	$0.875 \times 8 = 7$	7
$44 \div 8 = 5.5$	$0.5 \times 8 = 4$	4
$5 \div 8 = 0.628$	$0.625 \times 8 = 5$	5

- *To convert an octal number to binary:*
  - Simply replace each octal digit with the appropriate three bits.
  - For example: convert 142_8 to binary.
    Solution: $142_8 = 001\,100\,010$.
- *The conversion of a binary-to-octal number is the reverse of the octal-to-binary conversion.*
  - For example: convert 001 111 101 to octal number.
    Solution: $001\,111\,101 = 175_8$

Exercise 3.2.2 Convert the following octal numbers to decimal:

1. $54_8 =$ ____________________

2. $206_8 =$ ____________________

3.3 Miscellaneous Exercises

Exercise 3.3.1

1. *Evaluate* $2\cos\frac{\pi}{5}$ *correct to three significant figures.*

2. *Factorise* $3x^2 + x - 2$.

3. *Simplify* $\frac{2}{x} - \frac{1}{x+1}$.

4. *Solve* $|4x - 3| = 7$.

5. *Expand and simplify* $(\sqrt{3} - 1)(2\sqrt{3} + 5)$.

6. *Find the sum of the first 21 terms of the arithmetic series 3 + 5 + 7 + . . .*

Exercise 3.3.2

1. *Evaluate* $\sqrt{\pi^2 + 5}$ *correct to three decimal places.*

2. *Solve* $2x - 5 > -3$ *and graph the solution on a number line.*

3. *Rationalise the denominator of* $\frac{1}{\sqrt{3}-1}$.

4. *Find the limiting sum of the geometric series* $\frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \ldots$.

5. *Factorise* $2x^2 + 5x - 12$

6. *Find the equation of the line that passes through the point (-1, 3) and is perpendicular to* $2x + y + 4 = 0$.