Year 11 Math Homework

Student Name: ______________	**Grade**: ______________
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17 Topic 17 — Polynomials (Part 2)

17.1 The Remainder Theorem

Definition: If a polynomial $P(x)$ is divided by $(x - a)$
then the remainder can be found by evaluating $P(a)$

Example 17.1.1 Find the remainder when $x^4 + 3x^3 - 2x + 4$ **is divided by** $x + 4)$

Solution: *Use the Remainder Theorem and evaluate* $P(-4)$*:*
$P(-4) = (-4)^4 + 3(-4)^3 - 2(-4) + 4 = 256 + (-192) + 8 + 4 = 76$
$\therefore$ *Remainder is* 76

Exercise 17.1.1 Use the Remainder Theorem to find the remainder in each case:

1. $(x^3 - 2x^2 + 4x + 5) \div (x - 3)$

2. $(2x^3 - 4x^2 + x + 8) \div (x - 2)$

3. $(2x^4 - 5x^2 + 10x - 5) \div (x - 1)$

Example 17.1.2 When $x^3 + 3x^2 - 4x + 2$ is divided by (x-m) the remainder is 14. Find all the possible values of m.

Solution: *Let* $P(x) = x^3 + 3x^2 - 4x + 2, \Rightarrow P(m) = m^3 + 3m^2 - 4m + 2 = 14$

$\therefore P(m) = m^3 + 3m^2 - 4m - 12 = 0$

Try $P(1) = (1)^3 + 3(1)^2 - 4(1) - 12 = 1 + 3 - 4 - 12 = -12 \neq 0$ $(x-1)$ *is not a factor,*

Try $P(-1) = (-1)^3 + 3(-1)^2 - 4(-1) - 12 = -6 \neq 0$ $(x+1)$ *is not a factor,*

Try $P(2) = (2)^3 + 3(2)^2 - 4(2) - 12 = 8 + 12 - 8 - 12 = 0$ $(x-2)$ *is a factor,*

$$
\begin{array}{r|llll}
 & x^2 & +5x & +6 & \\
\hline
x-2) & x^3+3x^2 & -4x & +2 & \\
 & -x^3+2x^2 & & & \\
\hline
 & 5x^2 & -4x & & \\
 & -5x^2 & +10x & & \\
\hline
 & & 6x & +2 & \\
 & & -6x & +12 & \\
\hline
 & & & 14 &
\end{array}
$$

$\therefore x^3 + 3x^2 - 4x + 2 = (x-2)(x^2 + 5x + 6) + 14$

Now $x^2 + 5x + 6, \Rightarrow (x+3)(x+2) = 0, \Rightarrow x = -3$ *and* $x = -2$

$\therefore$ *the possible values of m are* $m = 2,\ m = -2$ *and* $m = -3$

Exercise 17.1.2

1. *When* $x^4 + 2x^3 + 3x + k$ *is divided by* $(x+3)$ *the remainder is 23. Find the value of m.*

2. *When* $x^2 - 5x + 4$ *is divided by (x-m) the remainder is* -2. *Find all possible values of m.*

3. *If* $x^3 - x^2 + mx + 6$ *has a remainder of 4 when divided by* $(x+2)$. *Find the value of m.*

17.2 The Factor Theorem

Definition: Consider any polynomial P(x) If P(a) = 0, then (x-a) is a factor of P(x)

Steps in factoring:

1. Find the first factor $(x - a)$ using a trial and error approach.
   Try $P(1),\ P(-1), P(2),\ P(-2)\,etc.$ Until the remainder is zero.
2. The other factor $Q(x)$ can then be found by dividing the polynomial $P(x)$ by the fist factor $(x - a)$.
3. If this other factor is a quadratic, then it can be simply factorised by inspection in the usual way.
4. If this factor $Q(x)$ is a polynomial of degree higher than 2, then steps (1) and (2) are repeated for this other factor.

Example 17.2.1 Find the factor of $P(x) = 4x^3 - 7x^2 - 3x + 2$. Hence find the zeros of the polynomial.

Solution: *Use the trial and error approach to find the first factor:*

Try $P(1) = 6(1)^3 - 11(1)^2 - 3(1) + 2 = -6 \neq 0 \;\Rightarrow\; (x - 1)$ *is not a factor,*

Try $P(-1) = 6(-1)^3 - 11(-1)^2 - 3(-1) + 2 = -12 \neq 0 \;\Rightarrow\; (x + 1)$ *is not a factor,*

Try $P(2) = 6(2)^3 - 11(2)^2 - 3(2) + 2 = 0 \;\Rightarrow\; (x - 2)$ *is a factor,*

Now divide $6x^3 - 11x^2 - 3x + 2$ *by* $x - 2$ *to find the other factor.*

$$\begin{array}{r|l} & 6x^2 + x - 1 \\ x-2 & 6x^3 - 11x^2 - 3x + 2 \\ & -6x^3 + 12x^2 \\ \hline & x^2 - 3x \\ & -x^2 + 2x \\ \hline & -x + 2 \\ & x - 2 \\ \hline & 0 \end{array}$$

$\therefore 6x^3 - 11x^2 - 3x + 2 = (x - 2)(6x^2 + x - 1)$

Now $6x^2 + x - 1$ *is a trinomial which can be easily factorising in the normal way*

by using the "cross swords" method: $\therefore 6x^3 - 11x^2 - 3x + 2 = (x - 2)(2x + 1)(3x - 1)$

To find the zeros of the polynomial, we solve P(x) = 0

$i.e.\;\; (x - 2)(2x + 1)(3x - 1) = 0$

$\therefore$ *Zeros are :* $x = 2,\;\; x = -\frac{1}{2}$ and $x = \frac{1}{3}$

Exercise 17.2.1 Find the factors of the following polynomials:

1. $3x^3 - 28x^2 + 43x + 42$

2. $2x^3 - x^2 - 25x - 12$

Exercise 17.2.2

1. $(x + 2)$ *is a factor of* $x^2 - x - k$. *find k.*

2. $(x - 4)$ *is a factor of* $x^3 - kx - 12$. *Find k.*

3. $(x + 3)$ *and*$(x + 4)$ *are factors of* $x^3 + ax^2 - bx - 24$. *Find a and b.*

4. -3 *and* 4 *are zeros of* $x^3 - 6x^2 - ax + b$. *Find a and b.*

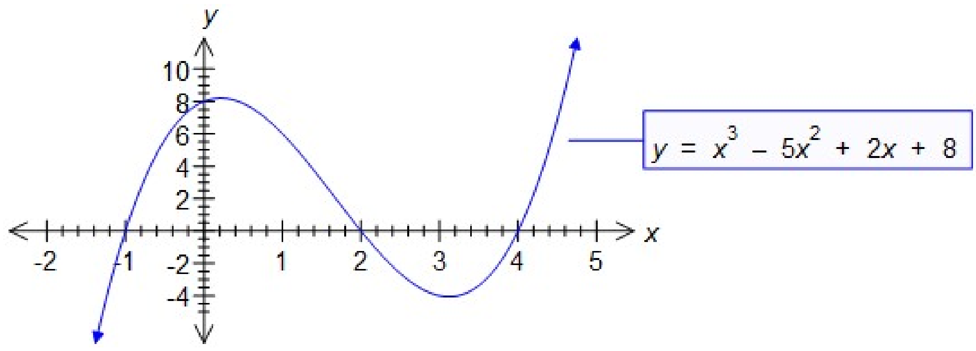
17.3 Using The Factor Theorem to Sketch Curves

Example 17.3.1 Use the Factor Theorem to find the zeros of $f(x) = x^3 - 5x^2 + 2x + 8$ and hence sketch the curve.

Solution: *Let $P(x) = x^3 - 5x^2 + 2x + 8$, and find the first factor by using trial and error:*
Try $P(1) = (1)^3 - 5(1)^2 + 2(1) + 8 = 1 - 5 + 2 + 8 = 6 \neq 0,\ (x-1)$ *is not a factor,*
Try $P(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 8 = -1 - 5 - 2 + 8 = 0,\ (x+1)$ *is a factor,*

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ \hline x+1 & x^3 - 5x^2 + 2x + 8 \\ & -x^3 - x^2 \\ \hline & -6x^2 + 2x \\ & 6x^2 + 6x \\ \hline & 8x + 8 \\ & -8x - 8 \\ \hline & 0 \end{array}$$

Therefore $f(x) = (x+1)(x^2 - 6x + 8)$
$\therefore f(x) = (x+1)(x-2)(x-4)$
To find the zeros, simplify solve $f(x) = 0$
$\therefore$ *zeros are* $x = -1,\ x = 2$ *and* $x = 4$
Also $f(x)$ *is a positive cubic and forms a "N" shape.*
Therefore final sketch is shown below:



Exercise 17.3.1 Sketch the following polynomials:

1. $y = -x(x+1)^2(x+3)$

2. $y = (x^2-1)(x+2)(x+4)$