Year 10 Term 3 Homework

Student Name: ____________	**Grade**: ____________
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6 Year 10 Term 3 Week 6 Homework

6.1 Functions and Logarithms

6.1.1 Functions and Inverse functions

- A function is a set of ordered pairs (x, y) in which for each x co-ordinate there is a unique y co-ordinate.
- If a graph can be cut by a vertical line in one point only, then the graph represents a function.
- If a graph can be cut by a vertical line in two or more points, then the graph does not represent a function.
- An inverse function is a function which reverses, or undoes another function.
- The graph of the inverse of a function is to reflect the graph of the function on the line $y = x$.
- The inverse of a function is a function if no horizontal line can be drawn to cut the graph of the original function more than once.

Exercise 6.1.1

1. *If* $f(x) = 2x^3$, *find the simplest form of* $\frac{f(x)-f(2)}{x-5}$.

2. *If* $g(x) = \frac{3+4x}{2-3x}$, *solve* $g(x) = -2$.

3. *If* $f(x) = \sqrt{x}$, *show that* $\frac{f(x)-f(1)}{x-1} = \frac{1}{1+\sqrt{x}}$.

Exercise 6.1.2 Find the inverse function for each of the following functions.

1. $y = \frac{5x}{5-x}$

2. $y = \frac{1}{x} + 3$

3. $y = \frac{1}{3}x - 1$

Exercise 6.1.3 Show that the function $f(x)$ and $g(x)$ are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

1. $f(x) = x^3 - 1$ *and* $g(x) = \sqrt[3]{x+1}$

2. $f(x) = \frac{x-2}{x+3}$*and* $g(x) = \frac{3x+2}{1-x}$

6.1.2 Translating graphs of functions

- The graph of $y = f(x) + k$. To sketch the graph of:
  1. $y = f(x) + k,\ (k > 0)$, shift the graph of $y = f(x)$ up k units.
  2. $y = f(x) - k,\ (k > 0)$, shift the graph of $y = f(x)$ down k units.
- The graph of $y = f(x - a)$. To sketch the graph:
  1. $y = f(x - a)$, (a>0), shift the graph $y = f(x)$ a units to the right.
  2. $y = f(x + a)$, (a>0), shift the graph $y = f(x)$ a units to the left.

Exercise 6.1.4

1. *Write down the equation of the new function if:*
   (a) $y = x^3 + 2$ *has been translated 5 units to the right.*
   (b) $y = \frac{2}{x}$ *has been translated 2 units up.*
   (c) $y = 2^x$ *has been translated 3 units down.*
2. *Find the new equation of each function after the given translations have occurred.*
   (a) $y = \frac{1}{3}x^2$ *has been translated up 2 units.*
   (b) $y = \frac{2}{3x}$ *has been translated right 2 units.*
   (c) $y = -\frac{1}{x}$ *has been translated down 4 units.*

6.1.3 Solving simple exponential equations

- The index laws: The following index laws will be used extensively in the the rest of this topic.

$a^m \times a^n = a^{m+n}$	$a^m \div a^n = a^{m-n}$	$(a^m)^n = a^{mn}$
$a^\circ = 1$	$a^{-1} = \frac{1}{a}$	$a^{-n} = \frac{1}{a^n}$
$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$	$\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$	$a^{\frac{1}{2}} = \sqrt{a}$
$a^{\frac{1}{3}} = \sqrt[3]{a}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$	$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

- Exponential equations. To solve an exponential equation:
  - express both sides of the equation with the same base
  - equate the indices
  - solve the resulting equation.
- Example: $4^x = 32$, $2^{2^x} = 2^5$; so $2x = 5$, and $x = \frac{5}{2}$

Exercise 6.1.5 Solve the following equations:

1. $36^x = 6$

2. $2^{2x+1} = 32$

3. $5^{x-5} = 1$

4. $27^{x-4} = 9^{4-x}$

5. $(\frac{1}{4})^{2-x} = \sqrt{8}$

6.2 Maths challenge

Exercise 6.2.1

1. *Prove that* $f(x) = 5x + 5$ *and* $g(x) = \frac{1}{5}x - 1$ *are inverse of each other.*

2. *Find the new equation of each function after the given translations have occurred.*

   (a) $y = \frac{1}{x^2}$

      i. *Up 5 units.*

      ii. *left 4 units.*

   (b) $y = 2^{-x}$

      i. *down 2 units.*

      ii. *right 3 units.*

3. *If* $f(x) = \frac{2}{x+1} + 1$, *Find the* $f^{-1}(x)$ *and* $f^{-1}(x - 1)$.