Year 10 Term 3 Homework

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

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This edition was printed on June 28, 2011.

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5 Year 10 Term 3 Week 5 Homework

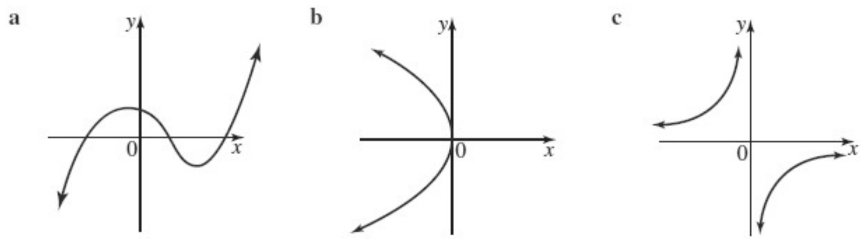
5.1 Functions and Logarithms

- A relation is any set of ordered pairs.
- A function is a set of ordered pairs (x, y) in which for each **x** co-ordinate there is a unique **y** co-ordinate.
- Every function is a relation, but not every relation is a function.

5.1.1 The vertical line test

- If a graph can be cut by a vertical line in one point only, then the graph represents a function.
- If a graph can be cut by a vertical line in two or more points, then the graph does not represent a function.

Example 5.1.1 State whether each graph represents a function.



Solution:

a *There is no vertical line that can be drawn to cut the graph more than once. Therefore the graph represents a function.*

b *It is possible to draw a vertical line that cuts the graph more than once. Therefore, the graph does not represent a function.*

c *The graph is discontinuous. However, it is not possible to draw a vertical line that cuts the graph more than once. Therefore, the graph represents a function.*

Exercise 5.1.1 Determine whether each of the following equations represents a function.

1. $y = 3x + 4$ ____________________
2. $y = \frac{1}{x}$ ____________________
3. $y = x^2$ ____________________
4. $x^2 + y^2 = 9$ ____________________

5.1.2 Function notation

- A function can be thought of as a machine. An input value is fed into the "the function machine" which produces an output value.
- The function has a one to one relationship.
- If $f(x) = 2x + 3$: then $f(0) = 2 \times 0 + 3 = 3$

Exercise 5.1.2 Find the values of each of the following functions:

1. *If* $f(x) = 3x + 5$, *find:*
   (a) $f(3)$ = ______
   (b) $f(-5)$ = ______
   (c) $f(0)$ = ______
2. *If* $g(x) = x^2 + 2x + 1$, *find:*
   (a) $g(5)$ = ______
   (b) $g(-2)$ = ______
   (c) $g(1)$ = ______
3. *If* $p(x) = x(2x - 3)$, *find:*
   (a) $p(2)$ = ______
   (b) $p(\frac{1}{2})$ = ______
   (c) $p(-2)$ = ______
4. *If* $q(x) = \frac{1}{x-1}$ *find:*
   (a) $q(\frac{1}{2})$ = ______
   (b) $q(2)$ = ______

Exercise 5.1.3

1. *If* $f(x) = 5x - 6$, *find the value of x for which* $f(x) = 24$.

2. *If* $g(x) = \sqrt{3x - 7}$, *solve* $g(x) = 2\sqrt{5}$.

3. *If* $h(x) = \frac{4+3x}{3-2x}$, *solve* $h(x) = -1$.

4. *If* $f(x) = x^2$, *find the simplest form of* $\frac{f(x)-f(2)}{x-2}$.

5.1.3 Inverse functions $y = f^{-1}(x)$

- An inverse operation reverses, or undoes another operation.
- An inverse function is a function which reverses, or undoes another function.
- The inverse of a function is found by reversing the values in each ordered pair.
- The inverse function of $y = 2x$ is $y = \frac{1}{2}x$.
- To graph the inverse of a function reflect the graph of the function in the line $y = x$.
- To find the equation of the inverse of a function:
  - interchange the variables x and y in the original equation.
  - make y the subject of the new equation.
- It does not necessarily follow that the inverse of a function is itself a function.
- The horizontal line test can be used to test if inverse equation is a function.
- If $f(x)$ is a function whose inverse is also a function, then the inverse function is written as $f^{-1}(x)$.

Exercise 5.1.4 Find the inverse function of each of these functions

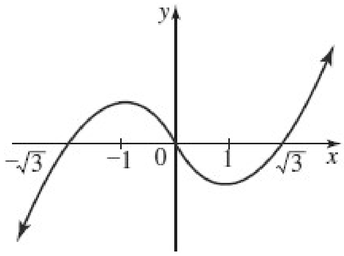
1. $y = \frac{x}{3} + 2$

2. $y = \frac{1}{x+5}$

3. $y = \frac{1}{x} - 3$

4. $y = \frac{x-2}{x+2}$

Exercise 5.1.5 The graph of $y = x^3 - 3x$ is shown.



1. *Is $y = x^3 - 3x$ a function?*

2. *Is the inverse of $y = x^3 - 3x$ a function?*

3. *Write down the largest possible set of permissible x value containing the origin such that the inverse function is also a function.*

Exercise 5.1.6 Write down a restricted set of permissible x-values, so that each function will have an inverse that is also a function.

1. $y = x^2$

2. $y = (x - 2)^2$

3. $y = x(x - 3)$

4. $y = (x - 1)^2 + 3$

5.2 Maths challenge

Exercise 5.2.1

1. *If* $f(x) = \frac{1}{x}$, *show that* $\frac{f(x)-f(1)}{x-1} = -\frac{1}{x}$.

2. $f(x) = \sqrt{x}$, *show that* $\frac{f(x)-f(1)}{x-1} = \frac{1}{1+\sqrt{x}}$.

3. *Find the inverses of* $y = \frac{3x}{5-x}$

4. *Find the inverse of* $y = \frac{7-2x}{x+2}$. *What do you notice?*

Exercise 5.2.2 Show that function $f(x)$ and g(x) are inverse of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

1. $f(x) = x + 5$ *and* $g(x) = x - 5$

2. $f(x) = 2x + 2$ *and* $g(x) = \frac{1}{2}x - 1$.

3. $f(x) = \frac{x-1}{x+2}$ *and* $g(x) = \frac{2x+1}{1-x}$.

5.3 Miscellaneous exercise

Exercise 5.3.1

1. *Solve* $\frac{x}{4} - \frac{3x-4}{3} = \frac{1}{3}$.

2. *Solve the inequality* $\frac{2}{3}(4x-3) \leq -(\frac{1}{6}(x-5)$.

3. *Simplify the following, expressing your answer as prime factors with positive indices.*

   (a) $\frac{2\times 4^{m+1}}{4\times 2^{m-1}}$

   (b) $\frac{25^{3m-2}}{125^{m-1}\times(5^{m+2})^2}$