Year 10 Term 2 Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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This edition was printed on June 24, 2014 with worked solutions.

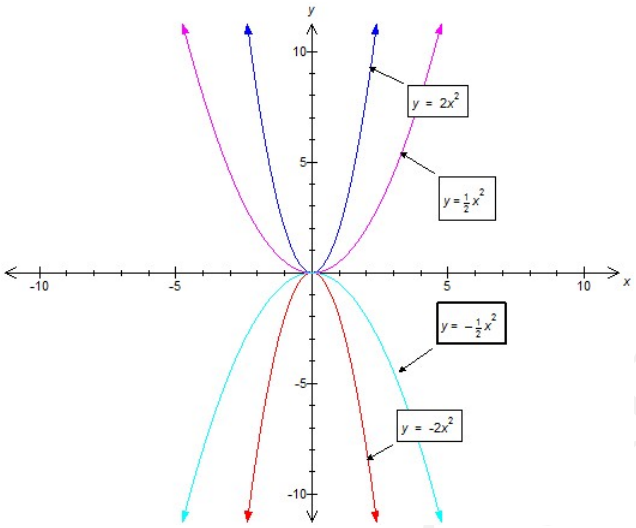
Camera ready copy was prepared with the LaTeX2e typesetting system.

4 Year 10 Term 2 Week 4 Homework

4.1 Graphs in the number plane

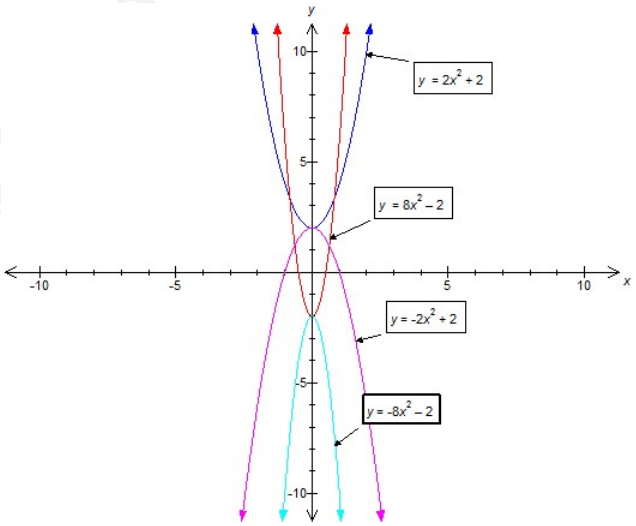
4.1.1 The parabola

- The graph of $y = ax^2$:



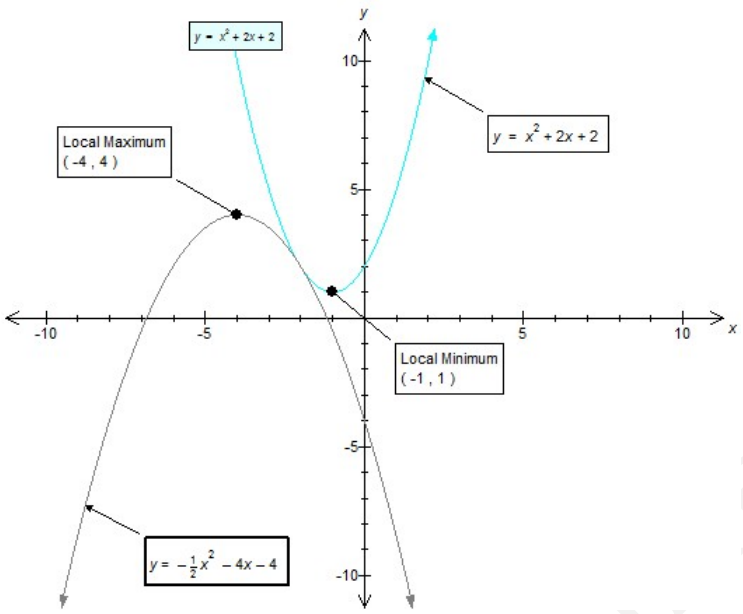
  - When $a > 0$ the curve is concave upward. When $a < 0$ the curve is concave downward.
  - The curve is symmetric about the y-axis.
  - The vertex has co-ordinates (0, 0).
  - The greater the value of a, (where $a > 0$), the narrower is the parabola.

- The graph of $y = ax^2 + c$:



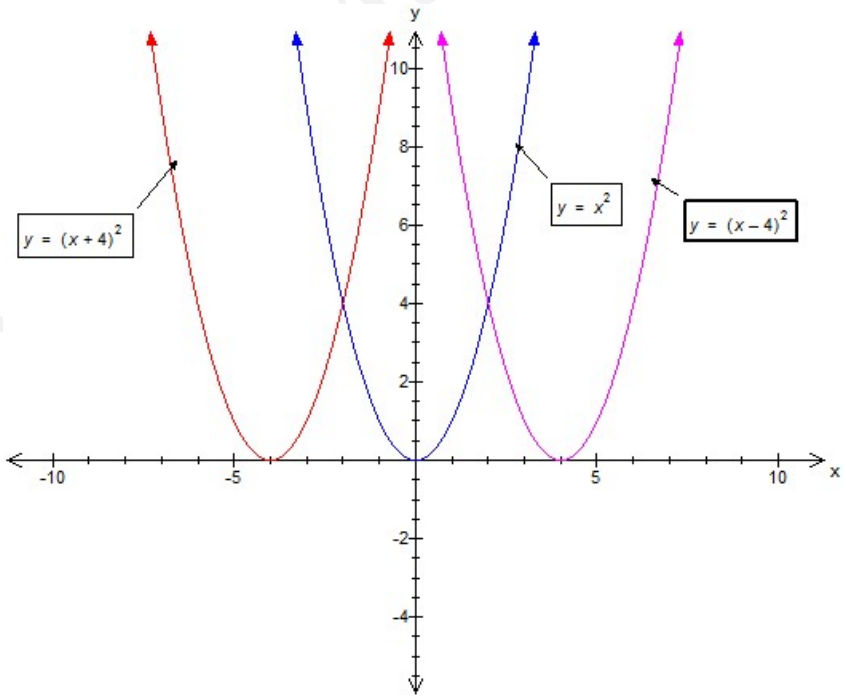
  - When $a > 0$ the curve is concave upward. When $a < 0$ the curve is concave downward.
  - The curve is symmetric about the y-axis.
  - The vertex has co-ordinates (0, c).
  - The greater the value of a, (where $a > 0$), the narrower is the parabola.

- The graph of $y = ax^2 + bx + c$:



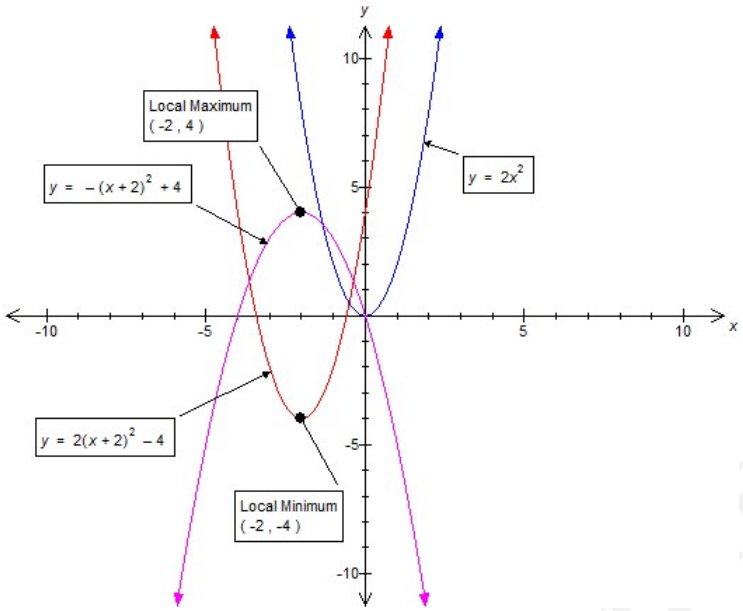
  - The equation of the axis of symmetry is $x = \frac{-b}{2a}$
  - When $a > 0$, the curve is concave down and the vertex is a minimum tuning point.
  - When the $a < 0$, the curve is concave up and the vertex is a maximum tuning point.

- The graph of $y = (x - h)^2$:



  - Shift the graph of $y = x^2$ to the right if $h > 0$. The vertex is in(h, 0).
  - Shift the graph of $y = x^2$ to the left if $h < 0$. The vertex is in (-h, 0)

- The graph of $y = a(x-h)^2 + k$



  - Determine whether the parabola is concave up (if $a > 0$) or concave down (if $a < 0$).
  - Shift the parabola $y = ax^2$ so that its vertex is (h, k).

Example 4.1.1 Express the equation $y = x^2 - 8x + 8$ in the form $y = (x-h)^2 + k$. Hence sketch the graph of $y = x^2 - 8x + 8$.

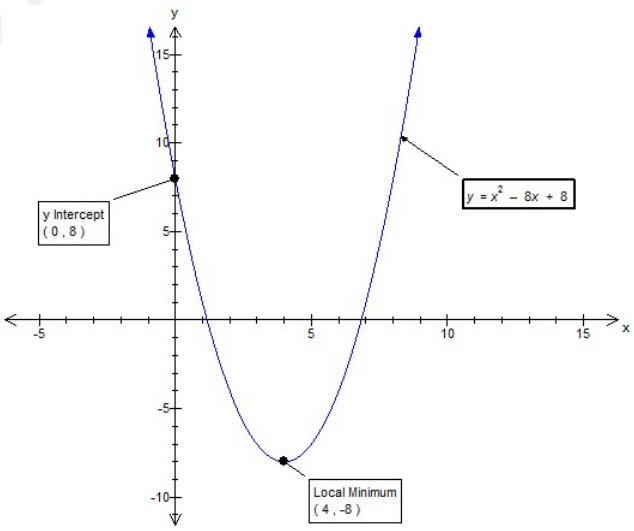
Solution: $y = x^2 - 8x + (\frac{8}{2})^2 + 8 - (\frac{8}{2})^2$

$y = (x^2 - 8x + 16) + 8 - 16$

$y = (x-4)^2 - 8$ *[∴ h = 4 and k = -8]*

∴ The co-efficient of the vertex are (4, -8), the parabola is concave up.

When x=0: $y = 0^2 + 8(0) + 8 = 9 \Rightarrow$ *∴ y-intercept is 8.*



4.1.2 Completing the square

Equation of the form $y = x^2 + bx + c$ can be expressed in the from $y = (x - h)^2 + k$ by completing the square.

- add $(\frac{b}{2})^2$ to $(x^2 + bx)$ to give $(x + \frac{b}{2})^2$.
- subtract $(\frac{b}{2})^2$ from c.

Equation of the form $y = ax^2 + bx + c$ can be expressed in the from $y = a(x - h)^2 + k$ by completing the square.

- add $a(\frac{b}{2a})^2$ to $ax^2 + bx$ to give $a(x + \frac{b}{2a})^2$.
- subtract $a(\frac{b}{2a})^2$ from c.

Exercise 4.1.1 Express each equation in the from $y = (x - h)^2 + k$, then state the co-ordinate of the vertex.

1. $y = x^2 + 6x + 8$

2. $y = x^2 - 8x - 4$

3. $y = 3x^2 + 12x + 4$

4.1.3 Proof of the quadratic formula

Example 4.1.2 Proof of the quadratic formula

Solution:

$$ax^2 + bx + c = 0$$
$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 = (\frac{b}{2a})^2 - \frac{c}{a}$$
$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$
$$\therefore\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4.1.4 Miscellaneous exercises

Exercise 4.1.2 Write each equation in the gradient-intercept form, then write down the gradient and y-intercept.

1. $3x + 4y - 12 = 0$

2. $3x - 2y + 6 = 0$

3. $2x - y + 13 = 0$

Exercise 4.1.3 Find the co-ordinates of the vertex for each of the following:

1. $y = 3x^2 - 6x + 2$

2. $y = 12 + 20x - 5x^2$

Exercise 4.1.4 For each of these parabolas find:

1. $y = 5 - 2x - 3x^2$

   (a) *y-intercept:*

   (b) *x-intercept:*

   (c) *axis of symmetry:*

   (d) *the vertex:*

   (e) *the maximum or minimum value:*

2. $y = 3x^2 - 5x - 2$

   (a) *y-intercept:*

   (b) *x-intercept:*

   (c) *axis of symmetry:*

   (d) *the vertex:*

   (e) *the maximum or minimum value:*

Exercise 4.1.5 Write each quadratic equation in the form of $f(x) = a(x-h)^2 + k$, find its x-intercept(s) if possible and then sketch the graph.

1. $f(x) = x^2 + 2x - 8$

2. $f(x) = 2x^2 - 8x + 1$

3. $f(x) = -2x^2 + 2x + 3$

Exercise 4.1.6 Find the domain and zeros of f(x).

1. $f(x) = \frac{5x^2}{x+3}$

2. $f(x) = \frac{-2(x^2-16)}{3(x^2-4x-5)}$

3. $f(x) = \frac{(x-1)(x^2+1)}{4x^3-x}$