Year 10 Term 1 Homework

Student Name: ______________	**Grade**: ______________
Date: ______________	**Score**: ______________

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6 Year 10 Term 1 Week 6 Homework

6.1 Triangle trigonometry

6.1.1 The tangent ratio

The tangent ratio can be expressed as the quotient of sine and cosine ratios. $tan\theta = \frac{sin\theta}{cos\theta}$

Exercise 6.1.1 Express each equation o terms of $tan\theta$, then solve for θ, correct to the nearest minute.

1. $12\cos\theta = 5\sin\theta$

2. $\frac{4}{\sin\theta} = \frac{1}{\cos\theta}$

3. $\frac{\sqrt{3}}{\sin\theta} = \frac{3}{\cos\theta}$

Exercise 6.1.2 Prove each of the following identities:

1. $\frac{\sin\theta\cos\theta}{\tan\theta} = \cos^2\theta$

2. $\frac{\cos\theta\tan\theta}{\sin\theta} = 1$

3. $\frac{\sin^2\theta+\sin\theta\cos\theta}{\cos^2\theta+\sin\theta\cos\theta} = \tan\theta$

6.1.2 The complementary results

- In any right-angled triangle, the sine of an acute angle is equal to the cosine of its complement,
- and the cosine of an acute angle is equal to the sine of its complement.
- $\sin\theta = \cos(90^\circ - \theta)$ and $\cos\theta = \sin(90^\circ - \theta)$

Example 6.1.1 Solve each of these equations:

1. $\cos(2x + 58)^\circ = \sin 12^\circ$

Solution:

$$\cos(2x + 58^\circ) = \cos(90^\circ - 12^\circ) = \cos 78^\circ,$$
$$\Rightarrow\ 2x + 58^\circ = 78^\circ$$
$$\Rightarrow \therefore\ x = 10^\circ.$$

2. $\sin(x + 18)^\circ = \cos(x - 18)^\circ$

Solution:

$$\sin(x + 18^\circ) = \sin[90^\circ - (x - 18^\circ)],$$
$$\Rightarrow\ x + 18^\circ = 90^\circ - x + 18^\circ,$$
$$2x = 90^\circ\ \Rightarrow \therefore\ x = 45^\circ.$$

Exercise 6.1.3 Simplify the following expressions:

1. $\frac{\cos(90^\circ - \theta)}{\tan\theta}$

2. $\sin(90^\circ - \theta)\cos(90^\circ - \theta)\tan(90^\circ - \theta)$

6.1.3 Trigonometric rations of obtuse angles

Definition: If θ is an acute angle, then:

- $\sin(180° - \theta) = \sin\theta$
- $\cos(180° - \theta) = -\cos\theta$
- $\tan(180° - \theta) = -\tan\theta$

Exercise 6.1.4 State whether the angle θ is acute or obtuse, where $0° < \theta° < 180°$, if:

1. $\sin\theta > 0$ *and* $\tan\theta > 0$ ______

2. $\cos\theta > 0$ *and* $\tan\theta > 0$ ______

3. $\tan\theta < 0$ *and* $\cos\theta < 0$ ______

Exercise 6.1.5 Express each of the following trigonometric ratios on terms of an acute angle, then evaluate, correct to 4 decimal places.

1. $\tan 129°$ ______

2. $\cos 152°$ ______

3. $\sin 135°$ ______

Exercise 6.1.6 For each of the following, find θ, where $0° < \theta° < 180°$. Answer correct to the nearest degree.

1. $\cos\theta = -0.208$ ______

2. $\tan\theta = -2.356$ ______

3. $\sin\theta = 0.848$ ______

Exercise 6.1.7 Prove each of the following identities:

1. $\frac{\sin\theta\sin(90^\circ-\theta)}{\cos\theta\cos(90^\circ-\theta)} = 1$

2. $\frac{\cos(180^\circ-\theta)\sin\theta}{\sin(180^\circ-\theta)} = -\cos\theta$

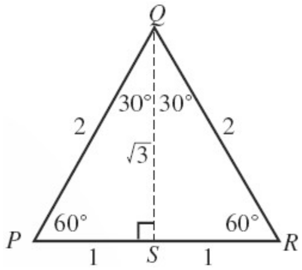
3. $\tan(180^\circ-\theta)\tan(90^\circ-\theta) = -1$

4. $\frac{\cos(90^\circ-\theta)}{\cos(180^\circ-\theta)} = -\tan\theta$

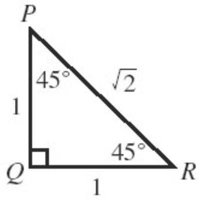
5. $\frac{\sin\theta\cos(90^\circ-\theta)}{\tan^2\theta} = \cos^2\theta$

6. $\tan(180^\circ-\theta)\sin(90^\circ-\theta) = -\sin\theta$

6.1.4 The exact values



θ	**30°**	**45°**	**60°**
sin θ	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$



Exercise 6.1.8 Find the exact value of each expression:

1. $\tan^2 60° - 2\cos^2 45°$

2. $2(\sin^2 45° + \cos^2 45°)$

3. $\sin^2 60° - \cos^2 60°$

Exercise 6.1.9 Show that:

1. $\sin 60° \cos 30° + \cos 60° \sin 30° = 1$

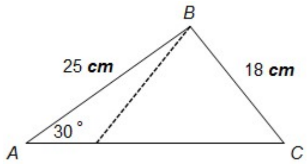
2. $\sin 45° \cos 60° + \cos 45° \sin 60° = \frac{\sqrt{6}+\sqrt{2}}{4}$

6.1.5 The Sine Rule

Definition: The Sine rule states that in any triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad or \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 6.1.2 I $\triangle ABC, \angle A = 30^\circ,\ BC = 18\,cm$ **and** $AB = 25\,cm.$ **Find the two possible values for** $\angle C$**, correct to nearest minutes. Hence show that there are two possible triangles.**



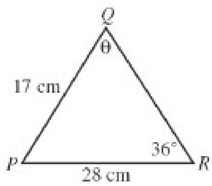
Solution: $\dfrac{\sin C}{25} = \dfrac{\sin 30^\circ}{18}, \Rightarrow \ \sin C = 25 \times \dfrac{\sin 30^\circ}{18} \ \Rightarrow \angle C = \sin^{-1} = 44^\circ.$

As $\sin(180^\circ - \theta) = \sin\theta. \ \Rightarrow \ \angle C = 180 - 44^\circ = 136^\circ.$

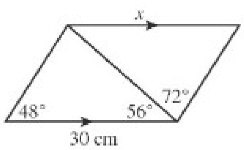
Therefore there are two possible triangles.

Exercise 6.1.10

1. *Find the size of the acute angle θ, correct to the nearest minute.*



2. *Find the value of x correct to 1 decimal place.*



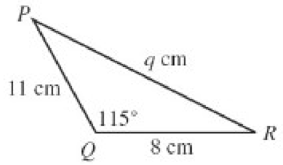
6.1.6 The Cosine Rule

Definition: The Cosine Rule states that in any triangle ABC:

$$a^2 = b^2 + c^2 - 2bc\cos A, \ \textit{and} \ \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Example 6.1.3

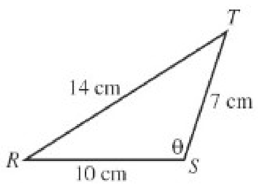
1. *Find the value of the pronumeral, correct to 1 decimal place.*



Solution: *From the formula:* $q^2 = 11^2 + 8^2 - 2(11)(8) \times \cos 115°$,

$$q^2 = 259.38, \Rightarrow \therefore \ q = \sqrt{259.38} = 16.1\,cm.$$

2. *Find the angle θ in the triangle given below, correct to the nearest minute.*



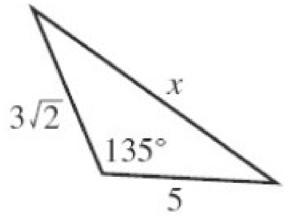
Solution: *From the formula:* $\cos\theta = \dfrac{10^2 + 7^2 - 14^2}{2(10)(7)} = -0.3357.$

$$\therefore \ \theta = 109°37'.$$

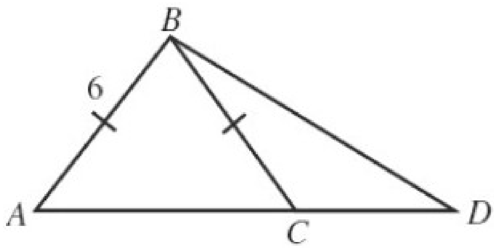
Exercise 6.1.11 The sides of a certain triangle are in the ratio $5 : 6 : 8$. Find the size of angles, correct to the nearest degree.

Exercise 6.1.12

1. *Find the exact value of x in the triangle shown below:*

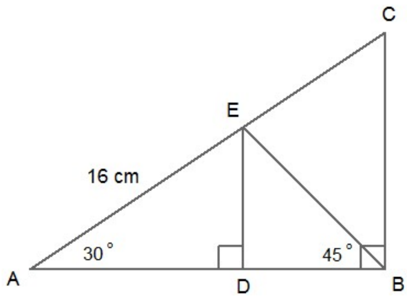


2. *In $\triangle ABC, AB = BC = 6$ cm. AC is produce to D so that $CD = 5$ cm and $BD = 9$ cm. Find the exact length of AC.*



6.2 Miscellaneous exercises

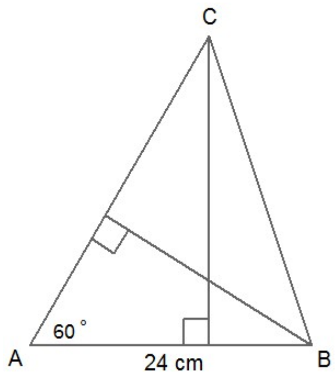
Example 6.2.1 Find the exact value of CE given that AE = 16 cm.



Solution:

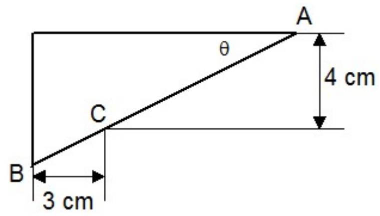
$$\sin 30^\circ = \frac{1}{2} = \frac{ED}{16} \Rightarrow ED = 8 = BD,$$
$$\cos 30^\circ = \frac{AD}{16}, \Rightarrow AD = 16 \times \frac{\sqrt{3}}{2} = 8\sqrt{3},$$
$$\therefore\ AB = 8 + 8\sqrt{3}$$
$$\cos 30^\circ = \frac{8 + 8\sqrt{3}}{16 + EC} = \frac{\sqrt{3}}{2} \Rightarrow \therefore\ EC = 16\frac{\sqrt{3}}{3}\ cm.$$

Exercise 6.2.1 In $\triangle ABC$, AB = 24 cm. $\angle CAB = 60^\circ$ and $\angle CBA = 75^\circ$. Find as exact values of AC and BC. Hence, find the area of $\triangle ABC$.



Exercise 6.2.2

1. *Find the distance AB shown in the figure (in exact value), given that* $\theta = 30°$.



2. *Simplify* $\frac{\frac{1}{x^2}}{1-\frac{1}{x}}$.

3. *Simplify* $\frac{5}{x-3} - \frac{x}{x^2-9}$.

4. *Evaluate* $(-8)^{\frac{2}{3}}$.