Year 10 Term 1 Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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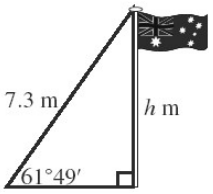
5 Year 10 Term 1 Week 5 Homework

5.1 Right-angle Triangle Trigonometry

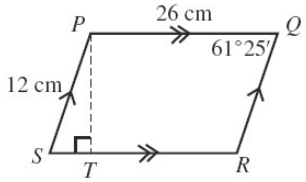
5.1.1 Find the length of a side

Exercise 5.1.1

1. *A wire of length 7.3 m attached to the top of a flagpole is inclined to the ground at an angle of* $61°49'$. *Find the height of the flagpole correct to the nearest centimetre.*



2. *In the parallelogram PQRS,* $PS = 12$ *cm,* $PQ = 26$ *cm and* $\angle PQR = 61°25'$.



   (a) *Find PT, the height of the parallelogram, correct to 1 decimal place.*

   (b) *Hence, calculate the area of the parallelogram.*

3. *From a window on the fifth floor of an office tower, a worker observes a ferry that has just moored across the street at Circular Quay. The window is 16.5 metres above the street and the angle of depression of the ferry from the window is* $24°40'$. *How far is the ferry from the base of the building? Answer correct to 4 significant figures.*

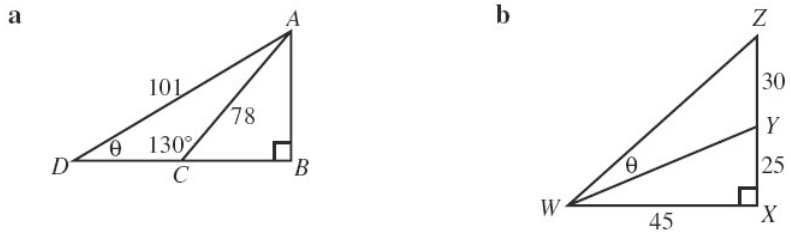
5.1.2 Find the size of an angle

Exercise 5.1.2

1. *A pendulum of length 85 cm swings through a horizontal distance of 60 cm before it stops and swings back again. Find the vertical angle through which the pendulum swings.*

2. *In $\triangle ABC$, $\angle B = 90°$, BC = 2AB and x is the midpoint of AB. Find the size of $\angle ACB$.*

3. *Find the angle θ in each of the following:*



   (a) *Find the size of θ for figure a:*

   (b) *Find the size of θ for figure b:*

4. *WXYZ is a quadrilateral in which $\angle X = 90°$, $\angle Z = 90°$, WX = 15 cm, XY = 20 cm, YZ = 24 cm and WZ = 7 cm. Find the size of $\angle W$.*

5.1.3 The tangent ratio

The tangent ratio can be expressed as the quotient of sine and cosine ratios. $tan\theta = \frac{sin\theta}{cos\theta}$

Exercise 5.1.3

1. *If* $\cos\theta = \frac{7}{25}$ *and* $\tan\theta = \frac{24}{7}$, *find the value of* $\sin\theta$.

2. *If* $\sin\theta = \frac{20}{29}$ *and* $\tan\theta = \frac{20}{21}$, *find the value of* $\cos\theta$.

3. *Solve the equation* $\sin\theta = \cos\theta$ *where* θ *is an acute angle.*

4. *In the equations below,* θ *is an acute angle. Express each equation in terms of* $tan\theta$, *hence solve for* θ, *correct to the nearest minute.*

   (a) $3\sin\theta = 5\cos\theta$

   (b) $12\cos\theta = 7\sin\theta$

   (c) $2.4cos\theta = 4.7sin\theta$

   (d) $\frac{1}{\sin\theta} = \frac{5}{\cos\theta}$

   (e) $\frac{\sqrt{5}}{\cos\theta} = \frac{2}{\sin\theta}$

5.1.4 The complementary results

- In any right-angled triangle,the sine of an acute angle is equal to the cosine of its complement,
- and the cosine of an acute angle is equal to the sine of its complement.
- $\sin\theta = \cos(90° - \theta)$ and $\cos\theta = \sin(90° - \theta)$

Example 5.1.1 Find the value of x in each of these following:

1. $\sin x° = \cos 80°$

   Solution: $\sin x° = \sin(90° - 80°)$
   $= \sin 10°$
   $\therefore x = 10°$

2. $\cos(2x)° = \sin 20°$

   Solution: $\cos(2x)° = \cos 70°$
   $2x = 70°$
   $\therefore x = 35°$

Exercise 5.1.4 Find the value of x in each of these:

1. $\sin 60° = \cos x°$ _____
2. $\cos 86° = \sin x°$ _____
3. $\sin x° = \cos 38°$ _____

Exercise 5.1.5 Simplify without using a calculator:

1. $\frac{\sin 30°}{3\cos 60°}$ _____
2. $\frac{2\cos 64°}{\sin 26°}$ _____
3. $\frac{3\cos 32°}{5\sin 58°}$ _____

Exercise 5.1.6

1. *If* $sin34° = 0.56$, *find the approximate value of* $sin34° + cos56°$

   _____

   _____

2. *If* $\cos 20° = 0.94$, *find the approximate value of* $5\sin 70°$

   _____

   _____

Exercise 5.1.7 Solve each of these equations:

1. $\sin(x + 20)^\circ = \cos 20^\circ$

2. $\sin(\frac{x}{4})^\circ = \cos 50^\circ$

3. $\cos(2x)^\circ = \sin(2x)^\circ$

4. $\cos(2x)^\circ = \sin x^\circ$

5. $\sin(2x + 36)^\circ = \cos 24^\circ$

6. $\cos(x + 28)^\circ = \sin(x - 18)^\circ$

7. $\cos(2x + 56)^\circ = \sin 14^\circ$

5.1.5 The exact values

Exercise 5.1.8

1. *Find the exact value of each of the following expressions. Give your answer in simplest surd form where necessary, with a rational denominator.*

   (a) $\frac{\tan 45^\circ}{\tan 30^\circ}$ = _____

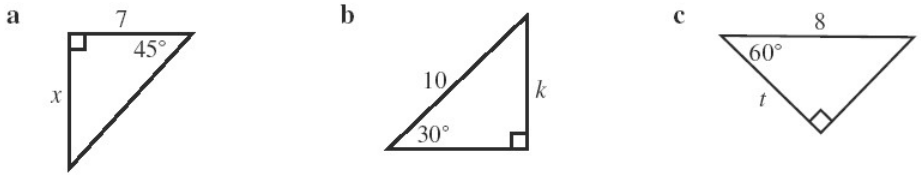
   (b) $\frac{\cos 45^\circ}{\cos 30^\circ}$ = _____

   (c) $\frac{sin30^\circ}{cos30^\circ}$ = _____

   (d) $\frac{\sin 30^\circ}{\cos 60^\circ}$ = _____

   (e) $\frac{\sin 60^\circ}{\cos 30^\circ}$ = _____

2. *Find the exact value of the pronumeral in each of the following. All lengths are in centimetres.*

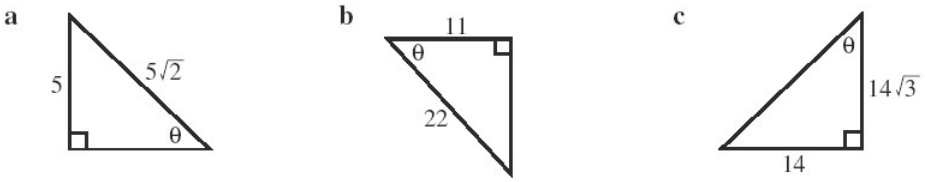


   (a) x = _____

   (b) k = _____

   (c) t = _____

3. *Find the size of the angle θ in each of these. All lengths are in centimetres.*



   (a) θ = _____

   (b) θ = _____

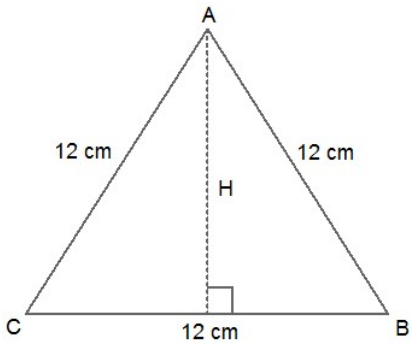
   (c) θ = _____

4. *If* $\tan\theta = 1$, *find the exact values for* $\sin\theta$. *From one of these ratios, calculate the value of* θ

5. *If* $\cos\beta = \frac{1}{2}$, *find the exact values of* $\sin\beta$ *and* $\tan\beta$.

6. *Show that* $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = 1$

7. *Triangle ABC is a equilateral triangle.*



   (a) *Use the exact values to find the height , H in centimetres.*

   (b) *Hence, find the exact area of the triangle.*

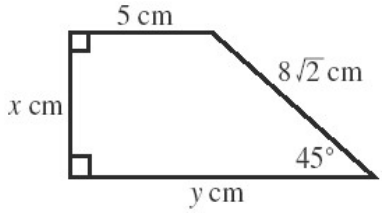
5.2 Miscellaneous Exercises

Exercise 5.2.1

1. *Given* $\cos\theta = \frac{3}{4}$*, derive expressions for* $\sin\theta$ *and* $\tan\theta$

2. *Given that* $\tan\alpha = \frac{5}{12}$*, find the value of* $\sin\alpha$ *and* $\cos\alpha$ *without calculate the* α*.*

3. *In the trapezium shown below:*



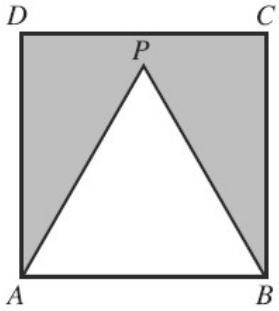
   (a) *Use the exact values to find the value of x.*

   (b) *Find value of y.*

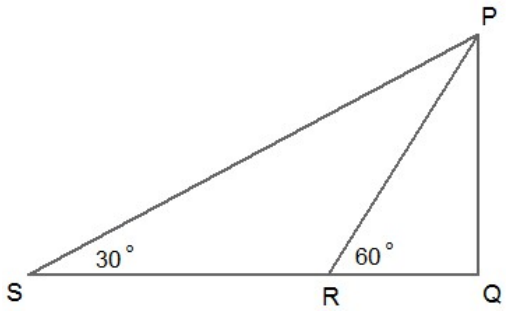
   (c) *Hence, find the area of the trapezium.*

Exercise 5.2.2

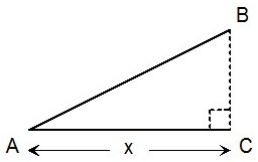
1. *ABCD is a square and APB is an equilateral triangle. If AB = 2 cm, find the area of the shaded area.*



2. *A man walked from S along level ground towards Q, the foot of a vertical cliff PQ. From S, the angle of elevation to the top of the cliff is* $30°$*. After walking to R, he noted that the angle of elevation was* $60°$ *If the height of the cliff is 100 metres, use the exact value to find the distance from R to S.*



Exercise 5.2.3 A wire of 10 cm is to be bent to form the hypotenuse and base of a right-angled triangle ABC, as shown in the diagram. Let the length of the base AC be x cm.



1. *What is the length of the hypotenuse AB in terms of x?*

2. *What is the height of the triangle in terms of x?*

3. *Hence find the area of the* $\triangle ABC$ *in terms of x.*

Exercise 5.2.4

1. *Find the exact value of* $\sin(\frac{\pi}{4}) + \sin(\frac{2\pi}{3})$

2. *By rationalising the denominator, express* $\frac{1}{2-\sqrt{3}} + \frac{1}{2+\sqrt{3}}$

3. *For what values of k does* $2x^2 + 3x + k = 0$ *have real roots?*

Exercise 5.2.5

1. *Consider a right angled triangle with angles A, B and C and opposite sides a, b, c respectively. If C is* $90°$*, find: (express answers in terms of a, b and c.)*

   (a) $\sin A$

   (b) $\tan B$

   (c) $\csc B$

2. *If c = 5 and b = 1, find:*

   (a) $\cos A$

   (b) $\cot A$

   (c) $\sec B$